# CP asymmetries in $\bar{B} \rightarrow \bar{K}^{*}(\rightarrow \bar{K} \pi) \bar{l} l$ and untagged $\bar{B}_{s}, B_{s} \rightarrow \phi\left(\rightarrow K^{+} K^{-}\right) \bar{l} l$ decays at NLO 

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AbStract: The decay $\bar{B} \rightarrow \bar{K}^{*}(\rightarrow \bar{K} \pi) \bar{l} l$ offers great opportunities to explore the physics at and above the electroweak scale by means of an angular analysis. We investigate the physics potential of the seven CP asymmetries plus the asymmetry in the rate, working at low dilepton mass using QCD factorization at next-to leading order (NLO). The $b \rightarrow s$ CP asymmetries are doubly Cabibbo-suppressed $\lesssim 1 \%$ in the Standard Model and its extensions where the CKM matrix is the only source of CP violation. Three CP asymmetries are T-odd, and can be $\mathcal{O}(1)$ in the presence of non-standard CP violation. The T-even asymmetries can reach $\mathcal{O}(0.1)$, limited by the small strong phases in the large recoil region. We furthermore point out an easy way to measure CP phases from time-integrated, untagged $\bar{B}_{d}, B_{d} \rightarrow K^{*}\left(\rightarrow K^{0} \pi^{0}\right) \overline{l l}$ and $\bar{B}_{s}, B_{s} \rightarrow \phi\left(\rightarrow K^{+} K^{-}\right) \bar{l} l$ decays. Analyses of these CP asymmetries can rule out, or further support the minimal description of CP violation through the CKM mechanism. Experimental studies are promising for (super) flavor factories and at hadron colliders.

Keywords: Rare Decays, Beyond Standard Model, B-Physics, CP violation.

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## 1. Introduction

The quest for physics beyond the Standard Model (SM) is of highest priority at current and future flavor facilities [1]. A promising direction is to look for CP symmetry breaking effects that cannot be accounted for with the SM's very predictive CKM-mechanism of CP and flavor violation for the quarks. Rare $b \rightarrow s$ transitions are such sensitive probes, since all CP violating effects in the SM receive a universal suppression of order $10^{-2}$ from the CKM matrix elements. Ongoing experimental efforts include those with hadronic decays such as $\bar{B}_{s}, B_{s} \rightarrow J / \Psi \phi$ at the Tevatron [2] and $b \rightarrow s \bar{s} s$ induced penguin modes, see, e.g., [3] for recent data.

We focus here on exclusive semileptonic decays induced by $b \rightarrow s \bar{l} l, l=e, \mu$, to explore the borders of the SM. The decays $\bar{B} \rightarrow \bar{K} \bar{l} l$ and $\bar{B} \rightarrow \bar{K}^{*} \bar{l} l$ have already been observed,
with their rate being in agreement with the SM. Investigations of more involved observables such as dilepton mass spectra, lepton angle distributions and dimuon to dielectron ratios are currently being pursued [目-[6]. The decay $\bar{B} \rightarrow \bar{K}^{*} \bar{l} l$ with subsequent $\bar{K}^{*} \rightarrow \bar{K} \pi$ allows to extract further physics information through an angular analysis of the $\bar{K} \pi \bar{l} l$ final state [7, especially on the chirality content [8- [10].

Here we study the eight CP asymmetries in $\bar{B} \rightarrow \bar{K}^{*}(\rightarrow \bar{K} \pi) \bar{l} l$ decays, the one in the decay rate plus seven more requiring angular information. We calculate the matrix elements to next-to-leading order (NLO) in the strong coupling and to lowest order (LO) in the $1 / E$ expansion [11], where $E$ denotes the energy of the emitted $\bar{K}^{*}$ in $\bar{B}$ rest frame. We work within QCD factorization (QCDF), which has been applied to $\bar{B} \rightarrow \bar{K}^{*} \bar{l} l$ decays 12, 13], for analyses in soft-collinear effective theory (SCET), see [14]. Previous works on CP asymmetries in the angular distributions (7, (15, 16] employed naive factorization, see (17) for a PQCD study.

By means of the large energy limit, the QCDF framework used in our analysis holds for small dilepton invariant masses, and predicts small strong phases: Lowest order quark loops are either close to or below threshold, hence real (charm quarks), or CKM suppressed or induced by small penguin contributions (the lighter quarks). Other sources of strong phases are from subleading spectator effects or higher order in $\alpha_{s}$.

This highlights T-odd CP asymmetries, three of which are accessible with the angular analysis. (The T-transformation reverses the sign of all particle momenta and spins.) The important feature is that the T-odd asymmetries are $\propto \cos \Delta_{S} \sin \Delta_{W}$, where $\Delta_{S}$ and $\Delta_{W}$ denote the differences of strong and weak phases, respectively. While the T-even CP asymmetries $\propto \sin \Delta_{S} \sin \Delta_{W}$ vanish for small strong phases, the T-odd asymmetries exhibit maximal sensitivity to CP violation in this limit.

Time-dependent CP asymmetries in $\bar{B}_{d}, B_{d} \rightarrow K^{*}\left(\rightarrow K^{0} \pi^{0}\right) \bar{l} l$ have also been considered in [16]. These measurements require large amounts of data. We suggest here to use the CP-odd property of four of the asymmetries to extract them from an untagged and time-integrated data set. It has been known that such data sets are useful to access CP violation in angular distributions, e.g., 18, 19]. We work out the corresponding CPsensitive observables in $\bar{B}_{d}, B_{d} \rightarrow K^{*}\left(\rightarrow K^{0} \pi^{0}\right) \bar{l} l$ and $\bar{B}_{s}, B_{s} \rightarrow \phi\left(\rightarrow K^{+} K^{-}\right) \bar{l} l$ decays. The $K^{+} K^{-} \bar{l} l$ final state is the same as the one of the $\bar{B}_{s}, B_{s} \rightarrow \phi\left(\rightarrow K^{+} K^{-}\right) J / \Psi(\rightarrow \bar{l})$ decays, which are already under experimental study including angular analysis [2].

In section 2 we review the $\bar{B} \rightarrow \bar{K}^{*}(\rightarrow \bar{K} \pi) \bar{l}$ angular distributions. CP asymmetries and possibilities of their measurement from double- and single-differential distributions are given in section 3. Prospects for $\bar{B}_{d}, B_{d} \rightarrow K^{*}\left(\rightarrow K^{0} \pi^{0}\right) \bar{l} l$ and $\bar{B}_{s}, B_{s} \rightarrow \phi\left(\rightarrow K^{+} K^{-}\right) \overline{l l}$ decays without tagging are examined in section 团. SM predictions and theoretical uncertainties are presented in section 5 . In section 6 we model-independently investigate the impact of New Physics (NP) on the CP asymmetries. To do so, we work out constraints from other rare decay data, $\bar{B} \rightarrow X_{s} \bar{l} l, X_{s} \gamma$, the forward-backward asymmetry in $\bar{B} \rightarrow \bar{K}^{*} \bar{l}$ and time-dependent CP asymmetries in $\bar{B}_{d}, B_{d} \rightarrow K^{*}\left(\rightarrow K^{0} \pi^{0}\right) \gamma$. We summarize in sec-


## 2. Full angular distribution

In this section we review the angular distribution of the exclusive decay $\bar{B} \rightarrow \bar{K}^{*}(\rightarrow \bar{K} \pi) \bar{l} l$ and its conjugate decay. Throughout this work we use $\bar{B} \equiv(b \bar{q})$ for $q=u, d, \bar{B}_{s} \equiv(b \bar{s})$ and $\bar{K}, \bar{K}^{*} \equiv(s \bar{q})$. We illustrate the kinematics for neutral mesons decaying to charged particles. Charged $B$-decays can be treated analogously. We discuss meson mixing effects and $\bar{B}_{s} \rightarrow \phi\left(\rightarrow K^{+} K^{-}\right) \bar{l} l$ decays in section $\#$.

The full angular distribution of the decay $\bar{B}^{0} \rightarrow \bar{K}^{* 0}\left(\rightarrow K^{-} \pi^{+}\right) \bar{l} l$ can be written in the limit of an on-shell decaying $K^{*}$ resonance as a 4 -differential distribution [7, [8]

$$
\begin{equation*}
\frac{d^{4} \Gamma}{d q^{2} d \cos \theta_{l} d \cos \theta_{K^{*}} d \phi}=\frac{3}{8 \pi} J\left(q^{2}, \theta_{l}, \theta_{K^{*}}, \phi\right), \tag{2.1}
\end{equation*}
$$

where the lepton spins have been summed over. Here, $q^{2}$ is the dilepton invariant mass squared, that is, $q^{\mu}$ is the sum of $p_{l^{+}}^{\mu}$ and $p_{l^{-}}^{\mu}$, the four momenta of the positively and negatively charged lepton, respectively. Furthermore, $\theta_{l}$ is defined as the angle between the negatively charged lepton and the $\bar{B}$ in the dilepton center of mass system (c.m.s.) and $\theta_{K^{*}}$ is the angle between the Kaon and the $\bar{B}$ in the $\left(K^{-} \pi^{+}\right)$c.m.s. . . We denote by $\mathbf{p}_{i}$ the three momentum vector of particle $i$ in the $\bar{B}$ rest frame. Then, $\phi$ is given by the angle between $\mathbf{p}_{K^{-}} \times \mathbf{p}_{\pi^{+}}$and $\mathbf{p}_{l^{-}} \times \mathbf{p}_{l^{+}}$, i.e., the angle between the normals of the ( $K^{-} \pi^{+}$) and $\left(l^{-} l^{+}\right)$planes. The full kinematically accessible phase space is bounded by

$$
\begin{equation*}
4 m_{l}^{2} \leqslant q^{2} \leqslant\left(M_{B}-M_{K^{*}}\right)^{2}, \quad-1 \leqslant \cos \theta_{l} \leqslant 1, \quad-1 \leqslant \cos \theta_{K^{*}} \leqslant 1, \quad 0 \leqslant \phi \leqslant 2 \pi, \tag{2.2}
\end{equation*}
$$

where $m_{l}, M_{B}$ and $M_{K^{*}}$ denote the mass of the lepton, $B$ meson and the $K^{*}$, respectively. For an off-resonance $\bar{B} \rightarrow \bar{K} \pi \bar{l} l$ study, see [20].

The dependence of the decay distribution (2.1) on the angles $\theta_{l}, \theta_{K^{*}}$ and $\phi$ can be made explicit as

$$
\begin{align*}
J\left(q^{2}, \theta_{l}, \theta_{K^{*}}, \phi\right)= & J_{1}^{s} \sin ^{2} \theta_{K^{*}}+J_{1}^{c} \cos ^{2} \theta_{K^{*}}+\left(J_{2}^{s} \sin ^{2} \theta_{K^{*}}+J_{2}^{c} \cos ^{2} \theta_{K^{*}}\right) \cos 2 \theta_{l} \\
& +J_{3} \sin ^{2} \theta_{K^{*}} \sin ^{2} \theta_{l} \cos 2 \phi+J_{4} \sin 2 \theta_{K^{*}} \sin 2 \theta_{l} \cos \phi+J_{5} \sin 2 \theta_{K^{*}} \sin \theta_{l} \cos \phi \\
& +J_{6} \sin ^{2} \theta_{K^{*}} \cos \theta_{l}+J_{7} \sin 2 \theta_{K^{*}} \sin \theta_{l} \sin \phi \\
& +J_{8} \sin 2 \theta_{K^{*}} \sin 2 \theta_{l} \sin \phi+J_{9} \sin ^{2} \theta_{K^{*}} \sin ^{2} \theta_{l} \sin 2 \phi, \tag{2.3}
\end{align*}
$$

where the coefficients $J_{i}^{(a)}=J_{i}^{(a)}\left(q^{2}\right)$ for $i=1, \ldots, 9$ and $a=s, c$ are functions of the dilepton mass. In the following we suppress the $q^{2}$-dependence for brevity also in expressions derived from the $J_{i}^{(a)}$.

The angular coefficients $J_{i}^{(a)}$ can be expressed through the $K^{*}$ transversity amplitudes $A_{i}\left(q^{2}\right)$ with $i=\{\perp, \|, 0, t\}$, see appendix A. Note that not all the $J_{i}^{(a)}$ are independent, for example, for vanishing lepton masses

$$
\begin{equation*}
3 J_{1}^{s}=J_{2}^{s}, \quad J_{1}^{c}=-J_{2}^{c} \tag{2.4}
\end{equation*}
$$

Furthermore, in the absence of right-handed currents $J_{3}=J_{9}=0$ up to power corrections.

The corresponding distribution of the CP conjugated decay $B^{0} \rightarrow K^{* 0}\left(\rightarrow K^{+} \pi^{-}\right) \bar{l} l$ can be written as

$$
\begin{equation*}
\frac{d^{4} \bar{\Gamma}}{d q^{2} d \cos \theta_{l} d \cos \theta_{K^{*}} d \phi}=\frac{3}{8 \pi} \bar{J}\left(q^{2}, \theta_{l}, \theta_{K^{*}}, \phi\right) \tag{2.5}
\end{equation*}
$$

Here, $\theta_{K^{*}}$ denotes the angle between the Kaon and the $B$ meson in the ( $K^{+} \pi^{-}$) c.m.s... The definiton of $\theta_{l}$ is identical for both $B$ and $\bar{B}$ decays. The angle $\phi$ for $B$ decays is given by the angle between $\mathbf{p}_{K^{+}} \times \mathbf{p}_{\pi^{-}}$and $\mathbf{p}_{l^{-}} \times \mathbf{p}_{l^{+}}$. Therefore, in the limit of unbroken CP, the distributions for $B$ and $\bar{B}$ mesons are equal under the combined transformations $\theta_{l} \rightarrow \theta_{l}-\pi$ and $\phi \rightarrow-\phi$. The function $\bar{J}$ is hence obtained from $J$ in (2.3) by the replacements

$$
\begin{equation*}
J_{1,2,3,4,7}^{(a)} \rightarrow \bar{J}_{1,2,3,4,7}^{(a)}, \quad \quad J_{5,6,8,9} \rightarrow-\bar{J}_{5,6,8,9}, \tag{2.6}
\end{equation*}
$$

where $\bar{J}_{i}^{(a)}$ equals $J_{i}^{(a)}$ with all weak phases being conjugated [7].
With its rich multi-dimensional structure, the angular distributions in (2.1) and (2.5) have sensitivity to various effects modifying the SM, such as CP violation beyond CKM and/or right-handed currents. Given sufficient data, all $J_{i}^{(a)}$ and $\bar{J}_{i}^{(a)}$ can in principle be completely measured from the full angular distribution in all three angles $\theta_{l}, \theta_{K^{*}}$ and $\phi$.

The familiar dilepton invariant mass spectrum for $\bar{B} \rightarrow \bar{K}^{*} \bar{l}$ decays can be recovered after integration over all angles as

$$
\begin{equation*}
\frac{d \Gamma}{d q^{2}}=J_{1}-\frac{J_{2}}{3}, \quad \text { where } \quad J_{1,2} \equiv 2 J_{1,2}^{s}+J_{1,2}^{c} . \tag{2.7}
\end{equation*}
$$

The (normalized) forward-backward asymmetry $A_{\mathrm{FB}}$ is given after full $\phi$ and $\theta_{K^{*}}$ integration as ${ }^{1}$

$$
\begin{equation*}
A_{\mathrm{FB}}\left(q^{2}\right) \equiv\left[\int_{0}^{1}-\int_{-1}^{0}\right] d \cos \theta_{l} \frac{d^{2} \Gamma}{d q^{2} d \cos \theta_{l}} / \frac{d \Gamma}{d q^{2}}=J_{6} / \frac{d \Gamma}{d q^{2}} . \tag{2.8}
\end{equation*}
$$

By $d \bar{\Gamma} / d q^{2}$ and $\bar{A}_{\mathrm{FB}}\left(q^{2}\right)$ we refer to the corresponding spectra of the CP conjugated decays.

## 3. CP asymmetries

CP violating effects in the angular distribution are signaled by non-vanishing differences between the ( $q^{2}$-dependent) angular coefficients

$$
\begin{equation*}
\Delta J_{i}^{(a)}=\Delta J_{i}^{(a)}\left(q^{2}\right) \equiv J_{i}^{(a)}-\bar{J}_{i}^{(a)} . \tag{3.1}
\end{equation*}
$$

Of particular importance are the asymmetries related to the coefficients $J_{7,8,9}$. These are odd under $\phi \rightarrow-\phi$, and hence induce T -odd asymmetries $\Delta J_{7,8,9}$ which are not suppressed by small strong phases as predicted from QCDF.

The CP asymmetry in the dilepton mass distribution is commonly defined as, see (2.7),

$$
\begin{equation*}
A_{\mathrm{CP}}\left(q^{2}\right) \equiv \frac{d(\Gamma-\bar{\Gamma})}{d q^{2}} / \frac{d(\Gamma+\bar{\Gamma})}{d q^{2}}=\frac{1}{N_{\Gamma}}\left[\Delta J_{1}-\frac{\Delta J_{2}}{3}\right], \quad N_{\Gamma}=N_{\Gamma}\left(q^{2}\right)=\frac{d(\Gamma+\bar{\Gamma})}{d q^{2}} . \tag{3.2}
\end{equation*}
$$

[^0]Following [7], we define in addition to $A_{\mathrm{CP}}$ seven normalized CP asymmetries as

$$
\begin{equation*}
A_{i}\left(q^{2}\right) \equiv \frac{2 \Delta J_{i}}{N_{\Gamma}} \text { for } i=3,6,9, \quad A_{i}^{D}\left(q^{2}\right) \equiv-\frac{2 \Delta J_{i}}{N_{\Gamma}} \text { for } i=4,5,7,8 \tag{3.3}
\end{equation*}
$$

Note that up to differences in the normalization $A_{6}$ equals the forward-backward CP asymmetry $A_{\mathrm{FB}}^{\mathrm{CP}}$ advocated to search for non-standard CP violation in the decay $\bar{B} \rightarrow \bar{K}^{*} \overline{l l}$ 22, [23), see (2.8). For $q^{2}$-integrated quantities we introduce the notation

$$
\begin{equation*}
\langle X\rangle=\int_{q_{\min }^{2}}^{q_{\max }^{2}} d q^{2} X\left(q^{2}\right) \tag{3.4}
\end{equation*}
$$

where the integration from $q_{\min }^{2}$ to $q_{\max }^{2}$ should be in the low dilepton mass region in order to use the $1 / E$ expansion of QCD for theory predictions, see appendix Q . We then define the normalized $q^{2}$-integrated CP asymmetries as

$$
\begin{equation*}
\left\langle A_{i}\right\rangle \equiv 2 \frac{\left\langle\Delta J_{i}\right\rangle}{\left\langle N_{\Gamma}\right\rangle} \text { for } i=3,6,9, \quad\left\langle A_{i}^{D}\right\rangle \equiv-2 \frac{\left\langle\Delta J_{i}\right\rangle}{\left\langle N_{\Gamma}\right\rangle} \text { for } i=4,5,7,8 \tag{3.5}
\end{equation*}
$$

where the numerator and the denominator are integrated with the same $q^{2}$ cuts.
The CP asymmetries $\left\langle A_{i}\right\rangle(i=3,6,9)$ can, for example, be extracted from the doubledifferential distribution in $\theta_{l}$ and $\phi$,

$$
\begin{array}{r}
\frac{d^{2}\langle\Gamma\rangle}{d \cos \theta_{l} d \phi}=\frac{1}{4 \pi}\left\{\left\langle J_{1}\right\rangle+\left\langle J_{2}\right\rangle \cos 2 \theta_{l}+2\left\langle J_{3}\right\rangle \sin ^{2} \theta_{l} \cos 2 \phi\right. \\
\left.+2\left\langle J_{6}\right\rangle \cos \theta_{l}+2\left\langle J_{9}\right\rangle \sin ^{2} \theta_{l} \sin 2 \phi\right\} \tag{3.6}
\end{array}
$$

which is obtained from integrating (2.1) over $\theta_{K^{*}}$. After full $\theta_{l}$-integration follows

$$
\begin{equation*}
\frac{d\langle\Gamma\rangle}{d \phi}=\frac{1}{2 \pi}\left\{\left\langle J_{1}\right\rangle-\frac{\left\langle J_{2}\right\rangle}{3}+\frac{4}{3}\left\langle J_{3}\right\rangle \cos 2 \phi+\frac{4}{3}\left\langle J_{9}\right\rangle \sin 2 \phi\right\}, \tag{3.7}
\end{equation*}
$$

showing that $\left\langle\Delta J_{9}\right\rangle$ can be found from $d\langle\Gamma+\bar{\Gamma}\rangle / d \phi$, whereas $\left\langle\Delta J_{3}\right\rangle$ can be obtained from $d\langle\Gamma-\bar{\Gamma}\rangle / d \phi$, with $\left\langle\Delta J_{1}\right\rangle-\left\langle\Delta J_{2}\right\rangle / 3$ from $A_{\mathrm{CP}}$ without angular study, see (3.2).

The measurement of the CP asymmetries $\left\langle A_{i}^{D}\right\rangle(i=4,5,7,8)$ requires binning into $\cos \theta_{K^{*}}$ as

$$
\begin{align*}
\frac{d^{2}\left\langle A_{\theta_{K^{*}}}\right\rangle}{d \cos \theta_{l} d \phi} & \equiv\left[\int_{0}^{1}-\int_{-1}^{0}\right] d \cos \theta_{K^{*}} \frac{d^{3}\langle\Gamma\rangle}{d \cos \theta_{K^{*}} d \cos \theta_{l} d \phi}  \tag{3.8}\\
& =\frac{1}{2 \pi}\left\{\left\langle J_{4}\right\rangle \sin 2 \theta_{l} \cos \phi+\left\langle J_{5}\right\rangle \sin \theta_{l} \cos \phi+\left\langle J_{7}\right\rangle \sin \theta_{l} \sin \phi+\left\langle J_{8}\right\rangle \sin 2 \theta_{l} \sin \phi\right\} .
\end{align*}
$$

From here follows upon full $\theta_{l}$-integration

$$
\begin{equation*}
\frac{d\left\langle A_{\theta_{K^{*}}}\right\rangle}{d \phi}=\frac{1}{4}\left\{\left\langle J_{5}\right\rangle \cos \phi+\left\langle J_{7}\right\rangle \sin \phi\right\} . \tag{3.9}
\end{equation*}
$$

We learn that $\left\langle\Delta J_{5}\right\rangle$ can be extracted from $d\left\langle A_{\theta_{K^{*}}}+\bar{A}_{\theta_{K^{*}}}\right\rangle / d \phi$ whereas $\left\langle\Delta J_{7}\right\rangle$ can be obtained from $d\left\langle A_{\theta_{K^{*}}}-\bar{A}_{\theta_{K^{*}}}\right\rangle / d \phi$. The double asymmetry in $\theta_{K^{*}}$ and $\theta_{l}$,

$$
\begin{equation*}
\frac{d\left\langle A_{\theta_{K^{*}}, \theta_{l}}\right\rangle}{d \phi} \equiv\left[\int_{0}^{1}-\int_{-1}^{0}\right] d \cos \theta_{l} \frac{d^{2}\left\langle A_{\theta_{K^{*}}}\right\rangle}{d \cos \theta_{l} d \phi}=\frac{2}{3 \pi}\left\{\left\langle J_{4}\right\rangle \cos \phi+\left\langle J_{8}\right\rangle \sin \phi\right\}, \tag{3.10}
\end{equation*}
$$

allows to obtain $\left\langle\Delta J_{4}\right\rangle$ from $d\left\langle A_{\theta_{K^{*}, \theta_{l}}}-\bar{A}_{\theta_{K^{*}}, \theta_{l}}\right\rangle / d \phi$, whereas $\left\langle\Delta J_{8}\right\rangle$ can be extracted from $d\left\langle A_{\theta_{K^{*}}, \theta_{l}}+\bar{A}_{\theta_{K^{*}}, \theta_{l}}\right\rangle / d \phi$.

The latter considerations demonstrate how the CP violating angular coefficients $\Delta J_{i}$ for $i=3,4,5,7,8,9$ can be extracted from distributions in the angle $\phi$. The quantity $\Delta J_{6}$ can be measured easiest from the $\cos \theta_{l}$-distribution, i.e., by adding the (numerators of the) forward-backward asymmetries $A_{\mathrm{FB}}$ and $\bar{A}_{\mathrm{FB}}$, see (2.8). Note that only $A_{3}, A_{6}$ and $A_{9}$ can be obtained from a genuinely single-differential distribution. $A_{9}$ is the only T-odd asymmetry with this property.

Analytical expressions for all CP asymmetries at NLO in terms of the short distance coefficients from the electroweak Hamiltonian in appendix $B$ are presented in appendix $D$. In particular, we include $\mathrm{NLO} \alpha_{s}$-corrections thus present in this work the first analyses of CP asymmetries in the $\bar{B} \rightarrow \bar{K}^{*}(\rightarrow \bar{K} \pi) \bar{l} l$ angular distributions at this order.

Corrections from $B_{d}-\bar{B}_{d}$ mixing to the time integrated CP asymmetries in flavor specific (self-tagging) final states are of the order $\left|A_{\mathrm{SL}}^{d}\right| \lesssim \mathcal{O}\left(10^{-3}\right)$ 24 and can be neglected. Here, $A_{\mathrm{SL}}^{d}$ denotes the semileptonic asymmetry into wrong sign leptons in the $B_{d}$-system.

## 4. CP asymmetries without tagging

With the $J_{i}, i=5,6,8,9$ being odd under CP, see (2.6), the corresponding CP asymmetries can be extracted from $d \Gamma+d \bar{\Gamma}$, i.e., without identifying the flavor of the initial $b$ quark. This feature is very useful for $\bar{B}_{d}, B_{d} \rightarrow K^{*}\left(\rightarrow K^{0} \pi^{0}\right) \bar{l} l$ and $\bar{B}_{s}, B_{s} \rightarrow \phi\left(\rightarrow K^{+} K^{-}\right) \bar{l} l$ decays, which unlike $\bar{B}_{d}, B_{d} \rightarrow K^{*}\left(\rightarrow K^{\mp} \pi^{ \pm}\right) \bar{l} l$ or charged $B$-decays, are not self-tagging. We focus here on $B_{s}$-decays to CP eigenstates, but the formalism equally applies to $B_{d}$-decays after the corresponding replacements.

Both $\bar{B}_{s}$ and $B_{s}$ angular distributions are described by the angles $\theta_{l}, \theta_{K^{*}}$ and $\phi$. These are defined in complete analogy with $\bar{B} \rightarrow \bar{K}^{*}(\rightarrow \bar{K} \pi) \bar{l} l$ decays, see section 2: $\theta_{l}$ is the angle between the negatively charged lepton and the $\bar{B}_{s} / B_{s}$ in the dilepton c.m.s., $\theta_{K^{*}}$ denotes the angle between the $K^{-}$and the $\bar{B}_{s} / B_{s}$ in the ( $K^{-} K^{+}$) c.m.s. and $\phi$ is the angle between $\mathbf{p}_{K^{-}} \times \mathbf{p}_{K^{+}}$and $\mathbf{p}_{l^{-}} \times \mathbf{p}_{l^{+}}$.

To account for mixing, time-dependent transversity amplitudes need to be introduced:

$$
\begin{equation*}
A_{a}(t) \equiv A\left(\bar{B}_{s}(t) \rightarrow \phi\left(\rightarrow K^{+} K^{-}\right)_{a} \bar{l}\right), \quad \bar{A}_{a}(t) \equiv A\left(B_{s}(t) \rightarrow \phi\left(\rightarrow K^{+} K^{-}\right)_{a} \bar{l} \bar{l}\right), \tag{4.1}
\end{equation*}
$$

where $A_{a}(t),\left(\bar{A}_{a}(t)\right)$ denotes the amplitude for a meson born at time $t=0$ as a $\bar{B}_{s},\left(B_{s}\right)$ decaying through the transversity amplitude $a=\perp, \|, 0$ at later times $t$. Here we use for brevity $A_{a}(t)$ for both $A_{a}^{L}(t)$ and $A_{a}^{R}(t)$. The formulae for the unmixed transversity amplitudes, i.e., the ones at $t=0$ can be taken from appendix $\square$ with the requisite replacements in masses and hadronic parameters and differences in the spectator effects given in [13] to account for the $\bar{B}_{s} \rightarrow \phi$ transitions.

Untagged rates $d \Gamma+d \bar{\Gamma}$ can then be written as $(a, b=\perp, \|, 0) \quad 18$

$$
\begin{align*}
\bar{A}_{b}^{*}(t) \bar{A}_{a}(t)+ & A_{b}^{*}(t) A_{a}(t)=\frac{1}{2} A\left(B_{s} \rightarrow \phi\left(\rightarrow K^{+} K^{-}\right)_{b} \bar{l} l\right)^{*} A\left(B_{s} \rightarrow \phi\left(\rightarrow K^{+} K^{-}\right)_{a} \bar{l} l\right) \\
& \times\left[\left(1+\eta_{a} \eta_{b} \xi_{b}^{*} \xi_{a}\right)\left(e^{-\Gamma_{L} t}+e^{-\Gamma_{H} t}\right)+\left(\eta_{b} \xi_{b}^{*}+\eta_{a} \xi_{a}\right)\left(e^{-\Gamma_{L} t}-e^{-\Gamma_{H} t}\right)\right] . \tag{4.2}
\end{align*}
$$

Here, $\eta_{\|, 0}=+1$ and $\eta_{\perp}=-1$ are the CP eigenvalues of the final state and $\Gamma_{L(H)}$ denotes the width of the lighter (heavier) mass eigenstate. We neglect CP violation in mixing, which is bounded by the semileptonic asymmetry in the $B_{s}$-system $\left|A_{\mathrm{SL}}^{s}\right| \lesssim \mathcal{O}\left(10^{-2}\right)$ [2]. Furthermore,

$$
\begin{equation*}
\xi_{a}=e^{-i \Phi_{M}} \frac{A\left(\bar{B}_{s} \rightarrow \phi\left(\rightarrow K^{+} K^{-}\right)_{a} \bar{l} l\right)}{A\left(\bar{B}_{s} \rightarrow \phi\left(\rightarrow K^{+} K^{-}\right)_{a} \bar{l} \bar{l}\right)\left(\delta_{W} \rightarrow-\delta_{W}\right)}, \tag{4.3}
\end{equation*}
$$

where ( $\delta_{W} \rightarrow-\delta_{W}$ ) implies the conjugation of all weak phases in the denominator and $\Phi_{M}$ denotes the phase of the $B_{s}-\bar{B}_{s}$ mixing. It is very small in the $\mathrm{SM}, \Phi_{M}^{\mathrm{SM}}=2 \arg \left(V_{t s}^{*} V_{t b}\right)$.

The CP asymmetries $\Delta J_{i}(t)=J_{i}(t)-\bar{J}_{i}(t), i=5,6,8,9$ are then obtained by taking the real and imaginary parts of (4.2), adding or subtracting $\bar{A}_{b}^{k *}(t) \bar{A}_{a}^{k}(t)+A_{b}^{k *}(t) A_{a}^{k}(t)$ for $k=L$ and $k=R$, and taking into account normalization factors depending on the angular coefficient $J_{i}$, see appendix A. After time-integration follows from (4.2)

$$
\begin{align*}
\int_{0}^{\infty} d t\left(\bar{A}_{b}^{*}(t) \bar{A}_{a}(t)+\right. & \left.A_{b}^{*}(t) A_{a}(t)\right)=A\left(B_{s} \rightarrow \phi\left(\rightarrow K^{+} K^{-}\right)_{b} \bar{l}\right)^{*} A\left(B_{s} \rightarrow \phi\left(\rightarrow K^{+} K^{-}\right)_{a} \bar{l}\right) \\
& \times \frac{1}{\Gamma\left(1-y^{2}\right)}\left[\left(1+\eta_{a} \eta_{b} \xi_{b}^{*} \xi_{a}\right)-y\left(\eta_{b} \xi_{b}^{*}+\eta_{a} \xi_{a}\right)\right], \quad y=\frac{\Delta \Gamma}{2 \Gamma} \tag{4.4}
\end{align*}
$$

where $\Gamma=\left(\Gamma_{L}+\Gamma_{H}\right) / 2$ and the width difference $\Delta \Gamma=\Gamma_{L}-\Gamma_{H}$.
This expression (4.4) becomes transparent if one neglects strong phases, where $\xi_{a}=$ $e^{-i\left(\Phi_{M}-2 \Phi_{a}\right)}, \Phi_{a} \equiv \arg \left(A\left(\bar{B}_{s} \rightarrow \phi\left(\rightarrow K^{+} K^{-}\right)_{a} \bar{l}\right)\right.$. For $a \neq b, \eta_{a}=-\eta_{b}$ we obtain

$$
\begin{gather*}
\int_{0}^{\infty} d t \operatorname{Im}\left(\bar{A}_{b}^{*}(t) \bar{A}_{a}(t)+A_{b}^{*}(t) A_{a}(t)\right)=\frac{2}{\Gamma\left(1-y^{2}\right)}\left|A\left(B_{s} \rightarrow \phi\left(\rightarrow K^{+} K^{-}\right)_{b} \bar{l}\right)\right| \\
\times\left|A\left(B_{s} \rightarrow \phi\left(\rightarrow K^{+} K^{-}\right)_{a} \bar{l} l\right)\right| \cdot\left[\sin \left(\Phi_{a}-\Phi_{b}\right)-y \eta_{a} \sin \left(\Phi_{M}-\Phi_{a}-\Phi_{b}\right)\right]  \tag{4.5}\\
\int_{0}^{\infty} d t \sum_{a}\left(\left|\bar{A}_{a}(t)\right|^{2}+\left|A_{a}(t)\right|^{2}\right)=\frac{2}{\Gamma\left(1-y^{2}\right)} \sum_{a}\left|A\left(B_{s} \rightarrow \phi\left(\rightarrow K^{+} K^{-}\right)_{a} \bar{l}\right)\right|^{2} \\
\times\left[1-y \eta_{a} \cos \left(\Phi_{M}-2 \Phi_{a}\right)\right], \tag{4.6}
\end{gather*}
$$

where (4.5) gives the asymmetries related to $J_{8,9}$ and (4.6) is needed for normalization, that is, gives (twice) the CP averaged decay rate. It also exhibits sensitivity to CP phases. The T-even asymmetries associated with $J_{5,6}$ vanish with no strong phases present.

We define ( $q^{2}$-dependent) CP-odd CP asymmetries $A_{i}^{(D) m i x}$ as $(a=\perp, \|, 0, k=L, R$ )

$$
\begin{align*}
A_{i}^{\text {mix }}\left(q^{2}\right) & \equiv 2 \frac{\int_{0}^{\infty} d t \Delta J_{i}(t)}{\int_{0}^{\infty} d t \sum_{a, k}\left(\left|\bar{A}_{a}^{k}(t)\right|^{2}+\left|A_{a}^{k}(t)\right|^{2}\right)} \\
A_{i}^{D m i x}\left(q^{2}\right) & \equiv-2 \frac{\text { for } \quad}{\int_{0}^{\infty} d t \sum_{a, k}\left(\left|t \Delta J_{a}^{k}(t)\right|^{2}+\left|A_{a}^{k}(t)\right|^{2}\right)} \quad \text { for } \quad i=6,9, \tag{4.7}
\end{align*}
$$

which match the CP asymmetries of the flavor-specific, unmixed decays (3.3) for $y \rightarrow 0$.
For $B_{d}$-mesons, $y$ is below $10^{-2}$ [24], and the untagged and time-integrated $K^{0} \pi^{0}$ final states yield the same information on CP violation as the ones with $K^{\mp} \pi^{ \pm}$discussed in section 3 , or charged $B$-decays. (For early works with $y=0$, see 19$]$ ). For $B_{s}$-mesons the width difference is larger, $y \sim \mathcal{O}(0.1)$ [24], and interference effects become observable with

| $\lambda=0.2258_{-0.0017}^{+0.0016} \quad$ (95\% C.L.) 26] | $\mathcal{B}\left(\bar{B} \rightarrow X_{c} l \bar{\nu}_{l}\right)=(10.57 \pm 0.15) \%$ |
| :---: | :---: |
| $\left\|V_{c b}\right\|=0.0417 \pm 0.0013$ (95\% C.L.) 26] | $\tau_{B^{0}}=(1.530 \pm 0.009) \mathrm{ps}$ |
| $\bar{\rho}=[0.108,0.243] \quad$ (95\% C.L.) [26] | $\tau_{B_{s}}=(1.425 \pm 0.041) \mathrm{ps}$ |
| $\bar{\eta}=[0.288,0.375] \quad$ (95\% C.L.) 26] | $f_{B_{u, d}}=(200 \pm 30) \mathrm{MeV}$ |
| $\alpha_{s}\left(m_{Z}\right)=0.1176 \pm 0.0020$ (24] | $f_{B_{s}}=(240 \pm 30) \mathrm{MeV}$ |
| $\alpha_{e}\left(m_{b}\right)=1 / 133$ | $\lambda_{B,+}(1.5 \mathrm{GeV})=(0.458 \pm 0.115) \mathrm{GeV}$ [13, 29 |
| $m_{W}=80.403 \mathrm{GeV}$ | $f_{\perp}^{K^{*}}(1 \mathrm{GeV})=(185 \pm 10) \mathrm{MeV}$ |
| $m_{t}^{\text {pole }}=(170.9 \pm 1.8) \mathrm{GeV}$ | $f_{\\|}^{K^{*}}=(217 \pm 5) \mathrm{MeV}$ |
| $m_{b}=(4.6 \pm 0.1) \mathrm{GeV}$ | $a_{1, K^{*}}^{\perp, \\|}(1 \mathrm{GeV})=0.1 \pm 0.07$ |
| $m_{c}^{\text {pole }}=(1.4 \pm 0.2) \mathrm{GeV}$ | $a_{2, K^{*}}^{\perp, \\|}(1 \mathrm{GeV})=0.1 \pm 0.1$ |

Table 1: The numerical input used in our analysis. We denote by $m_{b}$ the PS mass at the factorization scale $\mu_{f}=2 \mathrm{GeV}$. We neglect the strange quark mass throughout this work unless otherwise stated. The numerical input for the form factors $\xi_{\perp, \|}$ is given in appendix E.
the mixing phase $\Phi_{M}$. The latter is currently under intense experimental study and only poorly determined to date, see, e.g., [2].

We refrain in this work from presenting a dedicated numerical analysis for the $\bar{B}_{s}, B_{s} \rightarrow$ $\phi\left(\rightarrow K^{+} K^{-}\right) \bar{l} l$ observables $A_{i}^{(D) \text { mix }}$ : The presumably dominant part independent of the width difference can be inferred from $\bar{B} \rightarrow \bar{K}^{*}(\rightarrow \bar{K} \pi) \bar{l} l$ decays by $\operatorname{SU}(3)$. The biggest corrections such as those from the form factors and phase space are expected to cancel in the asymmetries. On the other hand, discrepancies in the CP asymmetries between $B_{d^{-}}$ and $B_{s}$-processes at $\mathcal{O}(y)$ can be attributed to the mixing parameters $y$ and $\Phi_{M}$.

## 5. Standard model predictions

CP asymmetries in the decays of hadrons are in the SM solely induced by the CKM matrix. For the $b \rightarrow s$ transitions under consideration here, the requisite weak phase difference stems from $\hat{\lambda}_{u}=V_{u b} V_{u s}^{*} / V_{t b} V_{t s}^{*}$. Therefore, all CP asymmetries in $\bar{B} \rightarrow \bar{K}^{*}(\rightarrow \bar{K} \pi) \bar{l} l$ decays discussed here receive an overall suppression by $\operatorname{Im}\left[\hat{\lambda}_{u}\right] \simeq \bar{\eta} \lambda^{2}$ of order $10^{-2}$, where $\lambda$ and $\bar{\eta}$ denote parameters of the Wolfenstein parametrization of the CKM matrix.

We work out the SM CP asymmetries in $\bar{B} \rightarrow \bar{K}^{*}(\rightarrow \bar{K} \pi) \bar{l} l$ decays in the low- $q^{2}$ region using QCDF at NLO in $\alpha_{s}$ and leading order $1 / E$. Analytical expressions for the asymmetries are given in appendix D. The CP asymmetries in the SM can be obtained by setting the NP Wilson coefficients $C_{7,9,10}^{(\prime), N P}=0$, see appendix $B$ for the effective Hamiltonian used. Details on the QCDF framework and the transversity amplitudes are given in appendix C. We take the $B \rightarrow K^{*}$ form factors from light cone QCD sum rules (LCSR) calculations (25], see appendix 因. Our numerical input is compiled in table 1. We checked that our findings for the branching ratio and the forward-backward asymmetry of $\bar{B} \rightarrow \bar{K}^{*} \bar{l}$ decays agree for the given input with [12, [13]. Our predictions always refer to neutral $B$-decays unless otherwise stated.

The three main uncertainties in the asymmetries come from the form factors $\xi_{\|}$and $\xi_{\perp}$, the variation of the renormalization scale $\mu_{b}$ and the CKM parameters. We vary the scale between $m_{b} / 2$ and $2 m_{b}$ and allow for an uncertainty of $11 \%$ and $14 \%$ for $\xi_{\perp}$ and $\xi_{\|}$, respectively. The CKM input is given in table 1]. For the total uncertainty estimate, all three sources of uncertainty are added in quadrature.

In figure 1 we show the T-odd CP asymmetries $A_{7,8}^{D}$ and the T-even ones $A_{\mathrm{CP}}, A_{4,5}^{D}$ and $A_{6}$ as a function of $q^{2}$. The various bands indicate the uncertainties due to the form factors, the CKM parameters, $\mu_{b}$ and the total uncertainty. The asymmetries $A_{3}$ and $A_{9}$ are not shown, since they vanish in the SM at lowest order in $1 / E$. (A small finite value is induced by the strange quark mass.) Hence, their leading contributions may arise as

$$
\begin{equation*}
A_{3,9} \sim \operatorname{Im}\left[\hat{\lambda}_{u}\right] \mathcal{O}(\Lambda / E) \sim \mathcal{O}\left(10^{-3}\right) \tag{5.1}
\end{equation*}
$$

The LO predictions for the CP asymmetries are also included in figure 1. The higher order $\alpha_{s}$-corrections increase the size of the CP asymmetries. For $A_{7}^{D}$ and $A_{8}^{D}$ this happens because their respective LO values are suppressed by cancellations. Specifically, in the SM

$$
\begin{equation*}
A_{7}^{D} \sim \operatorname{Im}\left[\hat{\lambda}_{u}\right] \operatorname{Re}\left[\frac{\mathcal{T}_{\perp}^{(u)}}{\xi_{\perp}}+\frac{q^{2}}{M_{B}^{2}} \frac{\mathcal{T}_{\|}^{(u)}}{\xi_{\|}}\right] \tag{5.2}
\end{equation*}
$$

which vanishes at LO in QCDF, see (C.4), and also (15). (Our value of $A_{7}^{D}$ at LO is tiny but finite since in the numerical analysis we do not neglect kinematical factors $M_{K^{*}}^{2} / M_{B}^{2}$.) The asymmetry $A_{8}^{D}$ is subject to similar cancellations, although here an additional LO term exists, which is, however, numerically subleading. The values of $A_{7}^{D}$ and $A_{8}^{D}$ are therefore determined by the $\mathrm{NLO} \alpha_{s}$-corrections resulting in a large $\mu_{b}$ uncertainty. The impact of the higher order terms on the T-even asymmetries is sizeable, but less pronounced. We discuss further details of the SM CP asymmetries in the context of the integrated CP asymmetries.

We find that all $q^{2}$-integrated CP asymmetries $\left\langle A_{i}^{(D)}\right\rangle$ are less than $\mathcal{O}\left(10^{-2}\right)$ in the SM. This can be seen in table 2, where we give the results for the two cuts $\left(q_{\min }^{2}, q_{\max }^{2}\right)=$ $(1,6) \mathrm{GeV}^{2}$ (upper entries) and $(1,7) \mathrm{GeV}^{2}$ (lower entries), respectively. The uncertainties from the form factors and $\mu_{b}$ are also shown separately. Due to the overall CKM factor $\operatorname{Im}\left[\hat{\lambda}_{u}\right]$, all CP asymmetries suffer from a universal $15 \%$ uncertainty related to that. The corresponding values for the NLO CP averaged decay rates are $\left\langle N_{\Gamma}\right\rangle / 2=$ $(1.1 \pm 0.3) \cdot 10^{-19} \mathrm{GeV}$ and $\left\langle N_{\Gamma}\right\rangle / 2=\left(1.3_{-0.3}^{+0.4}\right) \cdot 10^{-19} \mathrm{GeV}$ for $\left(q_{\text {min }}^{2}, q_{\max }^{2}\right)=(1,6) \mathrm{GeV}^{2}$ and $(1,7) \mathrm{GeV}^{2}$, respectively, assuming $\mathcal{B}\left(K^{*} \rightarrow K \pi\right)=100 \%$. The uncertainty in the rate is about $25 \%$ from the form factors, $7 \%$ from $V_{t s}$ and order one percent from $\mu_{b}$.

The form factor induced uncertainty in the asymmetries depends on the amount of cancellations between the numerator and the decay rate in the denominator. We recall that we vary the two form factors within their uncertainties independently. Taking into account correlations would reduce the errors in the ratios, but requires control over the parameters of the LCSR calculation [25], which is beyond the scope of this work. Since the decay rate is dominated by the longitudinal $K^{*}$ polarization driven by $\xi_{\|}$, see the discussion following (C.7), the strongest form factor uncertainty is seen in $A_{6}$ being proportional to


Figure 1: The CP asymmetries $A_{\mathrm{CP}, 6}\left(q^{2}\right)$ and $A_{4,5,7,8}^{D}\left(q^{2}\right)$ in the SM in the low- $q^{2}$ region at LO and NLO in QCDF. The various bands show the uncertainty due to the form factors, the CKM parameters and $\mu_{b}$ separately, whereas the overall band indicates the total uncertainty.
$\xi_{\perp}^{2}$. The other asymmetries $A_{i}^{D}, i=4,5,7,8$ with numerator $\propto \xi_{\|} \xi_{\perp}$ receive more efficient cancellations.

As can be seen from table $2,\left\langle A_{\mathrm{CP}}\right\rangle,\left\langle A_{7}^{D}\right\rangle$ and $\left\langle A_{8}^{D}\right\rangle$ exhibit a massive $\mu_{b}$ dependence of order $50 \%$. The CP asymmetries $A_{i}^{(D)}$ with $i=4,5,6$ are not subject to the cancellations mentioned after (5.2) and have a smaller residual $\mu_{b}$ uncertainty below ten percent. The $\mu_{b}$ dependence of $\left\langle A_{6}\right\rangle$ of a few percent is accidentally small due to significant cancellations between different $q^{2}$-regions, see the crossing of the $\mu_{b}$ bands in $A_{6}$ near $q^{2} \simeq(3-4) \mathrm{GeV}^{2}$ in figure 1 .

|  | SM $\cdot 10^{-3}$ | $\xi_{\perp, \\|}[\%]$ | $\mu_{b}[\%]$ | SM LO $\cdot 10^{-3}$ | $\operatorname{SM}\left(B^{\mp}\right) \cdot 10^{-3}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\left\langle A_{\mathrm{CP}}\right\rangle$ | $4.2_{-2.5}^{+1.7}$ | ${ }_{-24}^{+19}$ | ${ }_{-51}^{+33}$ | $3.0_{-1.5}^{+1.2}$ | $10.0_{-2.9}^{+2.3}$ |
|  | $4.8{ }_{-2.4}^{+1.7}$ | ${ }_{-17}^{+13}$ | +29 -44 | $3.1_{-1.6}^{+1.2}$ | $9.9{ }_{-2.8}^{+2.2}$ |
| $\left\langle A_{4}^{D}\right\rangle$ | $-1.8{ }_{-0.3}^{+0.3}$ | ${ }_{-8}^{+11}$ | ${ }_{-6}^{+2}$ | $-0.7_{-0.4}^{+0.4}$ | $-0.7_{-0.3}^{+0.4}$ |
|  | $-2.0_{-0.4}^{+0.4}$ | ${ }_{-8}^{+11}$ | +7 | $-0.8_{-0.4}^{+0.5}$ | $-1.1_{-0.4}^{+0.4}$ |
| $\left\langle A_{5}^{D}\right\rangle$ | $7.6_{-1.6}^{+1.5}$ | ${ }_{-13}^{+10}$ | ${ }_{-8}^{+7}$ | $2.7_{-1.2}^{+0.8}$ | $10.0_{-2.3}^{+2.2}$ |
|  | $7.6_{-1.6}^{+1.5}$ | ${ }_{-12}^{+9}$ | $\begin{gathered} +7 \\ { }_{-9} \end{gathered}$ | $2.7_{-1.2}^{+0.8}$ | $9.8{ }_{-2.1}^{+2.1}$ |
| $\left\langle A_{6}\right\rangle$ | $-6.4_{-2.7}^{+2.2}$ | ${ }_{-39}^{+31}$ | ${ }_{-2}^{+0}$ | $-1.9{ }_{-0.9}^{+1.0}$ | $-6.3_{-2.6}^{+2.1}$ |
|  | $-6.7_{-2.7}^{+2.2}$ | +30 +37 | ${ }_{+3}^{+1}$ | $-2.0_{-1.0}^{+1.1}$ | $-6.6_{-2.7}^{+2.2}$ |
| $\left\langle A_{7}^{D}\right\rangle$ | $-5.1_{-1.6}^{+2.4}$ | ${ }_{-8}^{+11}$ | ${ }_{-26}^{+42}$ | $<10^{-2}$ | $-7.1_{-1.9}^{+2.6}$ |
|  | $-4.6{ }_{-1.4}^{+2.1}$ | ${ }_{-6}^{+10}$ | ${ }_{-25}^{+42}$ | $<10$ | $-6.5_{-1.7}^{+2.3}$ |
| $\left\langle A_{8}^{D}\right\rangle$ | $3.5{ }_{-2.0}^{+1.4}$ | ${ }_{-10}^{+7.4}$ | ${ }_{-53}^{+37}$ | $0.2_{-0.08}^{+0.04}$ | $3.4{ }_{-2.0}^{+1.4}$ |
|  | $3.1_{-1.7}^{+1.3}$ | ${ }_{-10}^{+6}$ | ${ }_{-53}^{+37}$ | $0.14_{-0.06}^{+0.03}$ | $3.1_{-1.8}^{+1.3}$ |
| $\left\langle A_{3,9}\right\rangle^{\dagger}$ | $\mathcal{O}(1)$ |  |  | $\mathcal{O}(1)$ | $\mathcal{O}(1)$ |

Table 2: SM predictions for the integrated CP asymmetries in units of $10^{-3}$ with the integration boundaries $\left(q_{\min }^{2}, q_{\max }^{2}\right)=(1,6),(1,7) \mathrm{GeV}^{2}$ (from top to bottom). We take into account uncertainties from the form factors, the scale dependence $\mu_{b}$ and the CKM parameters, all of them added in quadrature for the total uncertainty. The relative uncertainties due to $\xi_{\perp}, \xi_{\|}$and $\mu_{b}$ are also shown separately. The asymmetries at LO in $\alpha_{s}$ and the NLO ones for charged $B$-decays are given as well, see text for details. ${ }^{\dagger}$ The leading contributions $\left\langle A_{3,9}\right\rangle$ in the SM are power counting estimates only.

Furthermore, we study the impact of higher order contributions in QCDF on the $\left\langle A_{i}^{(D)}\right\rangle$. The shift from LO in $\alpha_{s}$ to NLO is substantial. Switching off the spectator interactions reduces the size of $\left\langle A_{7,8}^{D}\right\rangle$ by about $10 \%$, and by less for the other asymmetries. We also give in table 2 the NLO SM predictions for charged $B$-decays. The splitting between the CP asymmetries in neutral versus charged $B$-decays is dominated by weak annihilation contributions from current-current operators and varies a lot in size: $\left\langle A_{5,7}^{D}\right\rangle$ $\left(\left\langle A_{\mathrm{CP}}\right\rangle\right)$ increase by $\mathcal{O}(30 \%$ ) (a factor of two) from neutral to charged $B$-decays, whereas $\left\langle A_{4}^{D}\right\rangle$ decreases by $\sim 1 / 2$. The splitting for $\left\langle A_{6,8}^{(D)}\right\rangle$ is at the few percent level.

The SM predictions for the untagged, time-integrated CP asymmetries $\left\langle A_{i}^{(D) m i x}\right\rangle, i=$ $5,6,8,9$, in $\bar{B}_{s}, B_{s} \rightarrow \phi\left(\rightarrow K^{+} K^{-}\right) \bar{l} l$ decays (4.7) can be inferred from the $\left\langle A_{i}^{(D)}\right\rangle$ in neutral $B_{d}$-decays, which are given in table 2. Corrections arise from $\operatorname{SU}(3)$ flavor breaking, which is expected to be small in the ratios, from the $B_{s}$-width difference at the level of ten percent and from spectator interactions. All these effects are subdominant with respect to the theoretical uncertainties of the SM predictions.

## 6. Beyond the standard model

This section contains the model-independent analysis of the CP asymmetries. We consider

| observable | SM | data |
| :--- | :---: | :--- |
| $\mathcal{B}\left(\bar{B} \rightarrow X_{s} \gamma\right)^{a}$ | $(3.15 \pm 0.23) \cdot 10^{-4}$ [34] | $(3.52 \pm 0.25) \cdot 10^{-4}$ |
| $S_{K^{*} \gamma}^{b}$ | $\left(-2.8_{-0.5}^{+0.4}\right) \cdot 10^{-2}$ | $-0.19 \pm 0.23$ |
| $\left.\mathcal{B}\left(\bar{B} \rightarrow X_{s} \bar{l} l\right)\right\|_{[1,6]}$ | $(1.59 \pm 0.11) \cdot 10^{-6}$ | $(35]$ |
| $\left.\mathcal{B}\left(\bar{B} \rightarrow X_{s} \bar{l} l\right)\right\|_{[>0.04]}$ | $(4.15 \pm 0.70) \cdot 10^{-6}$ | $(1.60 \pm 0.51) \cdot 10^{-6}$ |
| $\left\langle A_{\mathrm{FB}}\right\rangle_{\left[h i g h q^{2}\right]}^{c}$ | $<0$ | $(4.5 \pm 1.0) \cdot 10^{-6}$ |
| $\mathcal{B}\left(\bar{B}_{s} \rightarrow \bar{\mu} \mu\right)$ | $-\left(0.76_{-0.32}^{+0.52} \pm 0.07\right)$ |  |

Table 3: Relevant $b \rightarrow s \gamma$ and $b \rightarrow s \bar{l} l$ observables. ${ }^{a}$ With photon energy cut $E_{\gamma}>1.6 \mathrm{GeV}$. ${ }^{b} \mathrm{SM}$ value obtained with $m_{s}=0.12 \mathrm{GeV} .{ }^{c}$ Note the different lepton angle convention between [5, 6] and this work.

NP contributions to the operators $\mathcal{O}_{7,9,10}$ which are part of the effective Hamiltonian (B.1) of the SM, as well as NP contributions to the chirality flipped ones $\mathcal{O}_{7,9,10}^{\prime}$. We allow the respective NP coefficients $C_{i}^{\mathrm{NP}}$ and $C_{i}^{\prime \mathrm{NP}}=C_{i}^{\prime}$ for $i=7,9,10$ to vary in magnitude and phase, denoted by $\phi_{i}$, within the constraints from the FCNC $B$-decay data. The radiative and semileptonic $b \rightarrow s$ transitions are the most important ones for our analysis. The relevant data and SM predictions are given in table 3 .

In our analysis the NP Wilson coefficients are leading order coefficients. All Wilson coefficients are understood as evaluated at the low, $\mu_{b}$-scale. We start with a discussion of the experimental constraints.

### 6.1 Experimental constraints

The radiative decays induced by $b \rightarrow s \gamma$ probe the electromagnetic dipole coefficients $C_{7}$ and $C_{7}^{\prime}$. In the NP scenarios considered here, the flipped dipole coefficient has no interference terms in the radiative decay rates. Hence, these observables constrain only the magnitude of $C_{7}^{\prime}$ and not its phase.

We take into account the $\bar{B} \rightarrow X_{s} \gamma$ branching ratio for which we adopt the NNLO SM results from (34]. To account for the missing higher order calculation of the beyond-the-SM amplitude, we take for the theoretical uncertainty of the NP contribution twice the SM uncertainty. We apply the experimental constraints at $90 \%$ C.L. We checked that the direct CP asymmetry in $\bar{B} \rightarrow X_{s} \gamma$, e.g., [36], does not give constraints beyond those from the $\bar{B} \rightarrow X_{s} \gamma$ branching ratio.

The time-dependent CP asymmetry $S_{K^{*} \gamma}$ in $\bar{B}_{d}, B_{d} \rightarrow K^{* 0}\left(\rightarrow K^{0} \pi^{0}\right) \gamma$ [37] is important since it is sensitive to the interference of photons with different polarization, that is, photons coming from $\mathcal{O}_{7}$ versus $\mathcal{O}_{7}^{\prime}$. To illustrate the dependence on the Wilson coefficients, we give $S_{K^{*} \gamma}$ at lowest order (indicated by the superscript (0) for the contributions already present in the SM ):

$$
\begin{equation*}
S_{K^{*} \gamma}=-\frac{2|r|}{1+|r|^{2}} \sin \left(2 \beta-\arg \left(C_{7}^{(0)} C_{7}^{\prime}\right)\right), \quad r=C_{7}^{\prime} / C_{7}^{(0)} \tag{6.1}
\end{equation*}
$$

Here we assume that there is no beyond-the-SM physics in $B_{d}-\bar{B}_{d}$-mixing, and its phase is given by the CKM matrix elements. We calculate the exclusive $\bar{B} \rightarrow \bar{K}^{*} \gamma$ decay amplitude
with QCD factorization following [13] including $\alpha_{s}$-corrections. The constraints from $S_{K^{*} \gamma}$ exclude some regions with $|r|$ of order one, unless the CP phases conspire to suppress the sine in (6.1), see below.

The second class of constraints stems from the semileptonic transitions and applies to all Wilson coefficients we consider, $C_{7,9,10}^{\left({ }^{\prime}\right)}$. The inclusive $\bar{B} \rightarrow X_{s} \bar{l} l$ decays can be predicted with high accuracy, in the low- $q^{2}$ region at the level of $\lesssim 10 \%$ [35], but also the high $-q^{2}$ region is theoretically accessible. As can be seen in table 3, we utilize the integrated branching ratios in the low $-q^{2}$ region with $q^{2} \in[1,6] \mathrm{GeV}^{2},\left.\mathcal{B}\left(\bar{B} \rightarrow X_{s} \bar{l}\right)\right|_{[1,6]}$, as well as for $q^{2}>0.04 \mathrm{GeV}^{2},\left.\mathcal{B}\left(\bar{B} \rightarrow X_{s} \bar{l}\right)\right|_{[>0.04]}$. The latter has been experimentally obtained by cutting out events with $q^{2}$ close to the first and second charmonium resonance, hence bears some model-dependence. We use the corresponding theory predictions from 35] and [21], respectively. The decay distributions with NP are given in [38]. The treatment of uncertainties is as for the $\bar{B} \rightarrow X_{s} \gamma$ branching ratio.

Concerning the exclusive $\bar{B} \rightarrow \bar{K} \bar{l} l$ and $\bar{B} \rightarrow \bar{K}^{*} \bar{l}$ decays, we do not use the branching ratios for our model-independent analysis: the constraints are in general weaker than the ones from $\mathcal{B}\left(\bar{B} \rightarrow X_{s} \bar{l} l\right)$ due to the larger theoretical and experimental uncertainties. A particular difficulty with the available exclusive semileptonic decay data is the presence of measurements with different dilepton mass cuts, some of which in addition include regions where QCDF or SCET does not apply.

We employ instead early data on the $\bar{B} \rightarrow \bar{K}^{*} \bar{l} l$ forward-backward asymmetry from Belle and BaBar [ $\left[\begin{array}{|l}-6] \text {. While these measurements have large uncertainties, both experi- }\end{array}\right.$ ments strongly support the sign of $A_{\mathrm{FB}}$ in the high- $q^{2}$ region above the second charmonium peak to be SM-like.

A rigorous theory calculation of the exclusive $\bar{B} \rightarrow \bar{K}^{*} \bar{l}$ decays in this kinematical region can be facilitated with an operator product expansion in $\Lambda / Q$ and $m_{c}^{2} / Q^{2}$ where $Q=\left\{\sqrt{q^{2}}, m_{b}\right\}$ put forward in [39]. The leading contribution and also the order $m_{c}^{2} / Q^{2}$ terms do not introduce new non-perturbative matrix elements beyond naive factorization. Corrections start to enter at $\mathcal{O}\left(\alpha_{s} \Lambda / Q\right)$. The framework holds at low recoil, $\left(M_{B}-M_{K^{*}}\right)^{2}-$ $2 M_{B} \Lambda \lesssim q^{2}<\left(M_{B}-M_{K^{*}}\right)^{2}$, which covers the large dilepton mass region above the $\Psi^{\prime}$ resonance, $q^{2} \gtrsim 14 \mathrm{GeV}^{2}$.

To leading order in the $1 / Q$-expansion we obtain $A_{\mathrm{FB}}$ at low recoil as

$$
\begin{equation*}
A_{\mathrm{FB}}\left(q^{2}\right) \propto \operatorname{Re}\left[\left(C_{9}^{\mathrm{eff}}\left(q^{2}\right)+\frac{2 m_{b}^{2}}{q^{2}} C_{7}^{\mathrm{eff}}\right) C_{10}^{*}-\left(C_{9}^{\prime}+\frac{2 m_{b}^{2}}{q^{2}} C_{7}^{\prime}\right) C_{10}^{\prime *}\right] . \tag{6.2}
\end{equation*}
$$

The effective coefficients read as $C_{9}^{\text {eff }}\left(q^{2}\right)=C_{9}+\left(4 / 3 C_{1}+C_{2}\right) g\left(q^{2}\right)+\cdots$ and $C_{7}^{\text {eff }}=C_{7}+\cdots$, where $4 / 3 C_{1}+C_{2} \simeq 0.61$ are the dominant SM coefficients. The full expressions including the higher order $\alpha_{s}$-corrections and the QCD penguin contributions are given in [39] and are included in our numerical analysis. The lowest order charm loop function is given as

$$
\begin{equation*}
g\left(q^{2}\right)=\frac{8}{27}+\frac{4}{9}\left(\ln \frac{\mu^{2}}{q^{2}}+i \pi\right), \tag{6.3}
\end{equation*}
$$

which agrees with the perturbative quark loop function for massless quarks. Interestingly, the dependence on form factors can be factored out in $A_{\mathrm{FB}}(6.2)$ at this order. We require then the sign of $\left\langle A_{\mathrm{FB}}\right\rangle$ integrated over $q^{2}>14 \mathrm{GeV}^{2}$ to be negative.


Figure 2: Allowed regions of Wilson coefficients in specific NP scenarios after applying the experimental constraints from radiative and semileptonic $b \rightarrow s$ processes as indicated. The black areas are allowed by all constraints. In the left panel we show $C_{7}^{\prime}$ versus $C_{7}^{\mathrm{NP}}$ assuming real Wilson coefficients. We give the magnitude of $C_{10}^{\mathrm{NP}}$ versus its CP phase $\phi_{10}$ in the right panel. In both plots all other NP Wilson coefficients have been set to zero. For details see text.

We show the impact of the FCNC constraints on the NP Wilson coefficients for two NP scenarios in figure 2. The areas allowed by all constraints are given in black. We learn that the observables (each shown in a different color) yield complementary information, and that the SM is allowed, as well as many significantly different NP solutions.

In the left plot, we entertain NP only in $C_{7}$ and $C_{7}^{\prime}$, and assume further no NP CP phases. The regions allowed by $\mathcal{B}\left(\bar{B} \rightarrow X_{s} \gamma\right), S_{K^{*} \gamma}$ and $\left.\mathcal{B}\left(\bar{B} \rightarrow X_{s} \bar{l} l\right)\right|_{[1,6]}$ are shown as the green ring, the red cross and the blue half circle, respectively. The impact of $S_{K^{*} \gamma}$ is significant. The semileptonic decay excludes in this NP scenario the flipped sign solution for the photonic dipole coefficient $C_{7} \simeq-C_{7}^{\mathrm{SM}} \sim 0.31$. Note that dimensional analysis suggests that power corrections to $r$ of the order $C_{2} \Lambda /\left(3 m_{b} C_{7}\right) \sim 0.1$ may induce a larger SM contribution to $S_{K^{*} \gamma}$ than $\mathcal{O}\left(m_{s} / m_{b}\right) 40$. We show the resulting region in the $C_{7}^{\prime}-C_{7}^{\text {NP }}$-plane by the dashed lines in the left plot of figure 2. Because of the present experimental situation, however, the inclusion of the power corrections corresponds only to a small enlargement of the allowed parameter space. Note also that ref. 41] estimated the non-perturbative SM contributions to be smaller than the ones coming from naive power counting.

In the right plot we allow only for NP in $C_{10}$, and show the allowed regions in $\left|C_{10}^{\mathrm{NP}}\right|$ and the CP phase $\phi_{10}$. Fixing the sign of $A_{\text {FB }}$ (blue) and the semileptonic branching ratios $\left.\mathcal{B}\left(\bar{B} \rightarrow X_{s} \bar{l} l\right)\right|_{[1,6]}$ (green) and $\left.\mathcal{B}\left(\bar{B} \rightarrow X_{s} \bar{l}\right)\right|_{[>0.04]}$ (red) yield orthogonal constraints. An upper bound on the magnitude of $C_{10}^{\mathrm{NP}}$ is obtained with the aid of $A_{\mathrm{FB}}$ as $\left|C_{10}^{\mathrm{NP}}\right| \lesssim 7$, improving on the bound from the branching ratios alone, $\left|C_{10}^{\mathrm{NP}}\right| \lesssim 10$.

### 6.2 CP asymmetries with new physics

The dependence of the CP asymmetries $A_{i}^{(D)}$ on the Wilson coefficients can be seen from the analytical (NLO) formulae in appendix $D$. We also provide numerical model-independent

|  | generic NP | $C_{10}^{\text {NP }}$ only | $C_{10}^{\prime \text { NP }}$ only | $C_{9}^{\text {NP }}$ only |
| :---: | :---: | :---: | :---: | :---: |
| $\left\langle A_{\mathrm{CP}}\right\rangle$ | $[-0.12,0.10]$ | $[3,8] \cdot 10^{-3}$ | SM-like | $[-0.02,0.02]$ |
| $\left\langle A_{3}\right\rangle$ | $[-0.08,0.08]$ | SM-like | SM-like | SM-like |
| $\left\langle A_{4}^{D}\right\rangle$ | $[-0.04,0.04]$ | $[-4,-1] \cdot 10^{-3}$ | $[-3,-1] \cdot 10^{-3}$ | $[-0.01,0.01]$ |
| $\left\langle A_{5}^{D}\right\rangle$ | $[-0.07,0.07]$ | $[-0.04,0.04]$ | $[-0.02,0.04]$ | $[5,9] \cdot 10^{-3}$ |
| $\left\langle A_{6}\right\rangle$ | $[-0.13,0.11]$ | $[-0.05,0.05]$ | $[-9,-3] \cdot 10^{-3}$ | SM-like |
| $\left\langle A_{7}^{D}\right\rangle$ | $[-0.76,0.76]$ | $[-0.48,0.48]$ | $[-0.38,0.38]$ | SM-like |
| $\left\langle A_{8}^{D}\right\rangle$ | $[-0.48,0.48]$ | $[2,7] \cdot 10^{-3}$ | $[-0.28,0.28]$ | $[-0.17,0.17]$ |
| $\left\langle A_{9}\right\rangle$ | $[-0.62,0.60]$ | SM-like | $[-0.20,0.20]$ | SM-like |
| $\mathcal{B}\left(\bar{B}_{s} \rightarrow \bar{\mu} \mu\right)$ | $<1.4 \cdot 10^{-8}$ | $<6.3 \cdot 10^{-9}$ | $<1.3 \cdot 10^{-8}$ | SM |

Table 4: The ranges of the integrated CP asymmetries $\left\langle A_{i}^{(D)}\right\rangle$ for $\left(q_{\min }^{2}, q_{\max }^{2}\right)=(1,6) \mathrm{GeV}^{2}$ after applying the experimental constraints at $90 \%$ C.L. for the generic NP scenario and those with NP in $C_{10}, C_{10}^{\prime}$ or $C_{9}$ only. The upper limits on $\mathcal{B}\left(\bar{B}_{s} \rightarrow \bar{\mu} \mu\right)$ are also shown. For details see text.
formulae for the $\bar{B} \rightarrow \bar{K}^{*}(\rightarrow \bar{K} \pi) \bar{l} l$ branching ratio and CP asymmetries in appendix $\mathbb{E}$. The numerators of $A_{\mathrm{CP}, 3,4}^{(D)}$ are sensitive to $C_{7,9}$ and $C_{7,9}^{\prime}$ whereas the numerators of $A_{5,7}^{D}$ and $A_{6}$ probe $C_{7,10}$ and $C_{7,10}^{\prime}$. The numerators of $A_{8,9}^{(D)}$ can be affected by all Wilson coefficients considered here. Recall also that $A_{3,9}$ are very sensitive to the flipped Wilson coefficients since $A_{3,9}$ vanish in the limit $C_{i}^{\prime} \rightarrow 0$ at lowest order in the $1 / E$-expansion.

To see directly these features of the T-odd asymmetries, we provide LO formulae:

$$
\begin{align*}
& A_{7}^{D}=2 \mathcal{A}^{D} \frac{\hat{m}_{b}}{\hat{s}}(1-\hat{s}) \operatorname{Im}\left[\left(C_{10}^{(0)}-C_{10}^{\prime}\right)\left(C_{7}^{\mathrm{eff}(0)}-C_{7}^{\prime}\right)^{*}\right],  \tag{6.4}\\
& A_{8}^{D}=\mathcal{A}^{D} \beta_{l}\left\{\operatorname { I m } \left[C_{9}^{(0)} C_{9}^{\prime *}+C_{10}^{(0)} C_{10}^{\prime *}+\frac{4 \hat{m}_{b}^{2}}{\hat{s}} C_{7}^{\mathrm{eff}(0)} C_{7}^{\prime *}\right.\right. \\
& \left.+\frac{\hat{m}_{b}}{\hat{s}}\left((1-\hat{s})\left(C_{7}^{\prime} C_{9}^{\prime *}-C_{7}^{\mathrm{eff}(0)} C_{9}^{(0) *}\right)+(1+\hat{s})\left(C_{7}^{\mathrm{eff}(0)} C_{9}^{\prime *}-C_{7}^{\prime} C_{9}^{(0) *}\right)\right)\right] \\
& \left.-\operatorname{Re}\left(Y^{(0)}\right) \operatorname{Im}\left[C_{9}^{\prime}+\frac{\hat{m}_{b}}{\hat{s}}\left((1-\hat{s}) C_{7}^{\mathrm{eff}(0)}+(1+\hat{s}) C_{7}^{\prime}\right)\right]\right\}+\mathcal{O}\left(\hat{\lambda}_{u}\right),  \tag{6.5}\\
& A_{9}=4 \mathcal{A}^{D} \beta_{l}\left\{\operatorname{Im}\left[C_{9}^{(0)} C_{9}^{\prime *}+C_{10}^{(0)} C_{10}^{\prime *}+\frac{4 \hat{m}_{b}^{2}}{\hat{s}^{2}} C_{7}^{\mathrm{eff}(0)} C_{7}^{*}+\frac{2 \hat{m}_{b}}{\hat{s}}\left(C_{7}^{\mathrm{eff}(0)} C_{9}^{* *}-C_{7}^{\prime} C_{9}^{(0) *}\right)\right]\right. \\
& \left.-\frac{\hat{m}_{b}}{\hat{s}} \operatorname{Re}\left(Y^{(0)}\right) \operatorname{Im}\left[2 C_{7}^{\prime}+\frac{\hat{m}_{b}}{\hat{s}} C_{9}^{\prime}\right]\right\}+\mathcal{O}\left(\hat{\lambda}_{u}\right), \tag{6.6}
\end{align*}
$$

where for $A_{8}^{D}, A_{9}$ we neglected SM CP violation suppressed by $\hat{\lambda}_{u}$, see appendix D for details.

We work out the CP asymmetries $\left\langle A_{i}^{(D)}\right\rangle$ with NP by taking into account the experimental constraints given in table 3. We consider scenarios with generic NP, that is, when all six NP Wilson coefficients are varied independently, and when varying only one coefficient at a time. The asymmetries are integrated over low dilepton masses, $q^{2} \in[1,6] \mathrm{GeV}^{2}$. Theoretical input parameters used are fixed at their central values.


Figure 3: The dependence of the integrated T-odd CP asymmetries $\left\langle A_{7,8,9}^{(D)}\right\rangle$ for $\left(q_{\min }^{2}, q_{\max }^{2}\right)=$ $(1,6) \mathrm{GeV}^{2}$ on NP Wilson coefficients after applying the experimental constraints. In each plot all other NP Wilson coefficients have been set to zero.

In table we show the allowed ranges of the CP asymmetries in various NP scenarios. Numerically we find that the CP asymmetries can deviate significantly from their SM values, which are doubly Cabibbo-suppressed and below the percent level. As anticipated, the T-even CP asymmetries can be enhanced by one order of magnitude up to $\lesssim 10 \%$. The T-odd CP asymmetries $A_{7,8,9}^{(D)}$ can receive even stronger NP enhancements, up to order one.

There is also some residual dependence in the $A_{i}^{(D)}$ on all NP Wilson coefficients from the normalization to the CP averaged decay rate. Hence, even though the numerator of some CP asymmetries is independent of a particular Wilson coefficient, the asymmetries can be modified from their respective SM values given in table 2. These small effects are included in table whenever they are distinguishable from the SM at $1 \sigma$, otherwise called SM-like.

Also the purely leptonic decay $\bar{B}_{s} \rightarrow \bar{\mu} \mu$ has strong sensitivity to NP contributions in $\mathcal{O}_{10}$ and $\mathcal{O}_{10}^{\prime}$ since $\mathcal{B}\left(\bar{B}_{s} \rightarrow \bar{\mu} \mu\right) \propto\left|C_{10}-C_{10}^{\prime}\right|^{2}$, see, e.g., 22. We find a possible enhancement of $\mathcal{B}\left(\bar{B}_{s} \rightarrow \bar{\mu} \mu\right)$ up to almost an order of magnitude in NP scenarios with


Figure 4: Correlations between the CP asymmetries $\left\langle A_{7}^{D}\right\rangle$ and $\left\langle A_{6}\right\rangle$ in different scenarios of NP: generic NP, where all six coefficients are varied, and with NP in $C_{10}$ and $C_{7}$ only.
these coefficients modified, see table 7. The largest branching ratio, obtained with generic NP, is still a factor of two below the current experimental upper bound given in table 3. Furthermore, $\mathcal{B}\left(\bar{B}_{s} \rightarrow \bar{\mu} \mu\right)$ can be suppressed with respect to the SM by cancellations between $C_{10}$ and $C_{10}^{\prime}$. A lower bound exists from data on the decays $B \rightarrow K^{(*)} \bar{l}$, which are sensitive to $\left|C_{10}+C_{10}^{\prime}\right|$ [22]. However, in models containing both $C_{10}^{\mathrm{NP}}$ and $C_{10}^{\mathrm{NP}}$ only a very weak bound on $\mathcal{B}\left(\bar{B}_{s} \rightarrow \bar{\mu} \mu\right)$ can be obtained. We conclude that improved data on or a discovery of $\bar{B}_{s} \rightarrow \bar{\mu} \mu$ decays will have a strong impact on this type of analysis.

In figure 3 we show the dependence of the T-odd CP asymmetries $\left\langle A_{7,8,9}^{(D)}\right\rangle$ integrated over $\left(q_{\min }^{2}, q_{\max }^{2}\right)=(1,6) \mathrm{GeV}^{2}$ on the NP Wilson coefficients as indicated. In the plots all other NP Wilson coefficients have been set to zero and the experimental FCNC constraints have been applied. The dependence of the asymmetries on the phases is very strong, making the CP asymmetries great probes of CP violation beyond the SM.

In figure ${ }^{4}\left\langle A_{6}\right\rangle$ is shown against $\left\langle A_{7}^{D}\right\rangle$ for different NP scenarios. Both CP asymmetries are very sensitive to the phase of $C_{10}^{\left({ }^{\prime}\right)}$, and $A_{7}^{D}$ depends in addition on the dipole coefficients $C_{7}^{\left({ }^{\prime}\right)}$. Correlations of this type can identify the nature of NP.

## 7. Summary

We exploited the full angular analysis in exclusive semileptonic $\bar{B} \rightarrow \bar{K}^{*}(\rightarrow \bar{K} \pi) \bar{l} l$ and $\bar{B}_{s} \rightarrow \phi\left(\rightarrow K^{+} K^{-}\right) \bar{l} l$ decays as a means of testing the SM and searching for new CP phases in $b \rightarrow s$ transitions. From the angular distributions in (2.1) and (2.5) seven CP asymmetries $A_{3 \ldots 9}^{(D)}$ in addition to the one in the decay rate can be accessed. We find that the SM predictions, valid for low dilepton masses, have rather large uncertainties $\sim 20 \%$ for $A_{4,5}^{D}, \sim 50 \%$ for $A_{\mathrm{CP}}, A_{6}, A_{7,8}^{D}$ and order one for $A_{3,9}$, but the tiny magnitude of the CP asymmetries $\lesssim 10^{-2}$ makes them all ideal to search for a variety of different NP effects.

In particular, large NP effects are possible, which survive also the current experimental FCNC constraints. We summarize here specific features of the asymmetries:

- $A_{7}^{D}, A_{8}^{D}, A_{9}$ are T-odd and can be order one with New Physics.
- $A_{5}^{D}, A_{6}, A_{8}^{D}, A_{9}$ are CP-odd and can be obtained without tagging from $d \Gamma+d \bar{\Gamma}$.
- $A_{3}, A_{9}$ are very sensitive to right-handed currents.
- $A_{3}, A_{9},\left(A_{6}\right)$ can be extracted from a single-differential distribution in $\phi\left(\theta_{l}\right)$.
- $A_{7}^{D}$ is very sensitive to the phase of the $Z$-penguins $\sim C_{10}^{\left({ }^{( }\right)}$.

Due to the CP-odd feature of some of the asymmetries and the width difference in the $B_{s}$-system, further CP asymmetries $A_{i}^{(D) m i x}$ can be extracted from untagged, timeintegrated $\bar{B}_{s}, B_{s} \rightarrow \phi\left(\rightarrow K^{+} K^{-}\right) \bar{l} l$ decays. In the presence of NP with large CP phases we expect order one (order ten percent) CP asymmetries related to the T-odd (T-even) asymmetries $A_{8,9}^{(D) m i x}\left(A_{5,6}^{(D) m i x}\right)$. The difference between the asymmetries originating from $B_{d^{-}}$and $B_{s^{-}}$-decays is dominated by the difference in mixing.

A comparison of the $B_{d}$ and $B_{s}$ asymmetries hence probes the width difference and the mixing phase. One possibility is that the $A_{i}^{(D) m i x}$ are order 10 percent, whereas $A_{i}^{(D)}$ are negligible, indicating that there is beyond-the-SM CP violation in $\Delta B=2$ processes only. Note that a measurement of the $A_{i}^{(D) m i x}$ is probably the only easy way to study CP violation in semileptonic rare $b \rightarrow s$ transitions with $B_{s}$-mesons.

We conclude that the CP asymmetries from the angular analysis map out precisely the CP properties of several Wilson coefficients. The minimal, that is, CKM description of CP violation can be disproved. This study can be extended to include also lepton flavor dependent effects along the lines of (42].

The prospects for studying rare dimuon modes at the LHC are promising: For an integrated luminosity of $2 \mathrm{fb}^{-1}$, i.e., after one nominal year of data taking, a few thousand $B_{d} \rightarrow K^{* 0} \mu^{+} \mu^{-}$events are expected at LHCb, allowing a measurement of the branching ratio and its CP asymmetry at the percent level [43]. The CP asymmetries proposed here require further information on angular distributions, thus higher statistics. A dedicated sensitivity study, also taking into account suitable cuts, would be desireable.

## Acknowledgments

G.H. is happy to thank Frank Krüger for early discussions on $A_{7}^{D}$ and Robert Fleischer for careful reading of the manuscript. C.B. would like to thank Sebastian Jäger for helpful explanations. The work by C.B. and G.H. is supported in part by the Bundesministerium für Bildung und Forschung, Berlin-Bonn. G.P. is supported by a grant from the G.I.F., the German-Israeli-Foundation for Scientific Research and Development. G.H. is grateful to the organizers and participants of the Physics of the Large Hadron Collider-workshop and acknowledges the hospitality and great research environment provided by the Kavli Institute for Theoretical Physics in Santa Barbara during the final phase of this work.

## A. Angular coefficients $\boldsymbol{J}_{i}^{(a)}$

Here the functions $J_{i}^{(a)}$ in the angular distribution (2.1) are given in terms of the transversity amplitudes $A_{\perp, \|, 0, t}$ (9):

$$
\begin{align*}
& J_{1}^{s}=\frac{3}{4}\left\{\frac{\left(2+\beta_{l}^{2}\right)}{4}\left[\left|A_{\perp}^{L}\right|^{2}+\left|A_{\|}^{L}\right|^{2}+(L \rightarrow R)\right]+\frac{4 m_{l}^{2}}{q^{2}} \operatorname{Re}\left(A_{\perp}^{L} A_{\perp}^{R^{*}}+A_{\|}^{L} A_{\|}^{R^{*}}\right)\right\},  \tag{A.1}\\
& J_{1}^{c}=\frac{3}{4}\left\{\left|A_{0}^{L}\right|^{2}+\left|A_{0}^{R}\right|^{2}+\frac{4 m_{l}^{2}}{q^{2}}\left[\left|A_{t}\right|^{2}+2 \operatorname{Re}\left(A_{0}^{L} A_{0}^{R^{*}}\right)\right]\right\},  \tag{A.2}\\
& J_{2}^{s}=\frac{3 \beta_{l}^{2}}{16}\left[\left|A_{\perp}^{L}\right|^{2}+\left|A_{\|}^{L}\right|^{2}+(L \rightarrow R)\right],  \tag{A.3}\\
& J_{2}^{c}=-\frac{3 \beta_{l}^{2}}{4}\left[\left|A_{0}^{L}\right|^{2}+(L \rightarrow R)\right],  \tag{A.4}\\
& J_{3}=\frac{3}{8} \beta_{l}^{2}\left[\left|A_{\perp}^{L}\right|^{2}-\left|A_{\|}^{L}\right|^{2}+(L \rightarrow R)\right],  \tag{A.5}\\
& J_{4}=\frac{3}{4 \sqrt{2}} \beta_{l}^{2}\left[\operatorname{Re}\left(A_{0}^{L} A_{\|}^{L^{*}}\right)+(L \rightarrow R)\right],  \tag{A.6}\\
& J_{5}=\frac{3 \sqrt{2}}{4} \beta_{l}\left[\operatorname{Re}\left(A_{0}^{L} A_{\perp}^{L^{*}}\right)-(L \rightarrow R)\right],  \tag{A.7}\\
& J_{6}=\frac{3}{2} \beta_{l}\left[\operatorname{Re}\left(A_{\|}^{L} A_{\perp}^{L^{*}}\right)-(L \rightarrow R)\right],  \tag{A.8}\\
& J_{7}=\frac{3 \sqrt{2}}{4} \beta_{l}\left[\operatorname{Im}\left(A_{0}^{L} A_{\|}^{L^{*}}\right)-(L \rightarrow R)\right],  \tag{A.9}\\
& J_{8}=\frac{3}{4 \sqrt{2}} \beta_{l}^{2}\left[\operatorname{Im}\left(A_{0}^{L} A_{\perp}^{L^{*}}\right)+(L \rightarrow R)\right],  \tag{A.10}\\
& J_{9}=\frac{3}{4} \beta_{l}^{2}\left[\operatorname{Im}\left(A_{\|}^{L^{*}} A_{\perp}^{L}\right)+(L \rightarrow R)\right], \tag{A.11}
\end{align*}
$$

where

$$
\begin{equation*}
\beta_{l}=\sqrt{1-\frac{4 m_{l}^{2}}{q^{2}}} \tag{A.12}
\end{equation*}
$$

and the transversity amplitudes in QCDF can be seen in (C.5).

## B. The effective Hamiltonian

We use the $\Delta B=1$ effective Hamiltonian for $b \rightarrow s$ transitions, e.g., [44, (13]

$$
\begin{equation*}
\mathcal{H}_{\mathrm{eff}}=-\frac{4 G_{F}}{\sqrt{2}} V_{t b} V_{t s}^{*}\left(\mathcal{H}_{\mathrm{eff}}^{(t)}+\hat{\lambda}_{u} \mathcal{H}_{\mathrm{eff}}^{(u)}\right), \quad \quad \hat{\lambda}_{u}=V_{u b} V_{u s}^{*} / V_{t b} V_{t s}^{*}, \tag{B.1}
\end{equation*}
$$

where $V_{i j}$ denote CKM matrix elements and we used unitarity to write the basis as

$$
\begin{equation*}
\mathcal{H}_{\mathrm{eff}}^{(t)}=C_{1} \mathcal{O}_{1}^{c}+C_{2} \mathcal{O}_{2}^{c}+\sum_{i=3}^{10} C_{i} \mathcal{O}_{i}, \quad \mathcal{H}_{\mathrm{eff}}^{(u)}=C_{1}\left(\mathcal{O}_{1}^{c}-\mathcal{O}_{1}^{u}\right)+C_{2}\left(\mathcal{O}_{2}^{c}-\mathcal{O}_{2}^{u}\right) \tag{B.2}
\end{equation*}
$$

Here, the $\mathcal{O}_{1,2}^{u, c}$ denote current-current operators whereas $\mathcal{O}_{i}$ for $i=3,4,5,6$ are QCDpenguin operators, defined as in 45. We further take into account the following dipole and semileptonic operators

$$
\begin{align*}
\mathcal{O}_{7} & =\frac{e}{(4 \pi)^{2}} \bar{m}_{b}\left[\bar{s} \sigma^{\mu \nu} P_{R} b\right] F_{\mu \nu}, & \mathcal{O}_{7}^{\prime} & =\frac{e}{(4 \pi)^{2}} \bar{m}_{b}\left[\bar{s} \sigma^{\mu \nu} P_{L} b\right] F_{\mu \nu} \\
\mathcal{O}_{9} & =\frac{e^{2}}{(4 \pi)^{2}}\left[\bar{s} \gamma_{\mu} P_{L} b\right]\left[\bar{l} \gamma^{\mu} l\right], & \mathcal{O}_{9}^{\prime} & =\frac{e^{2}}{(4 \pi)^{2}}\left[\bar{s} \gamma_{\mu} P_{R} b\right]\left[\bar{l} \gamma^{\mu} l\right] \\
\mathcal{O}_{10} & =\frac{e^{2}}{(4 \pi)^{2}}\left[\bar{s} \gamma_{\mu} P_{L} b\right]\left[\bar{l}^{\mu} \gamma_{5} l\right], & \mathcal{O}_{10}^{\prime} & =\frac{e^{2}}{(4 \pi)^{2}}\left[\bar{s} \gamma_{\mu} P_{R} b\right]\left[\bar{l} \gamma^{\mu} \gamma_{5} l\right] \tag{B.3}
\end{align*}
$$

where $P_{R / L}=\left(1 \pm \gamma_{5}\right) / 2$ denote chiral projectors and $\bar{m}_{b}\left(\mu_{b}\right)$ is the $\overline{\mathrm{MS}} b$-quark mass at the scale $\mu_{b}$. Since in the SM $C_{i}^{\prime} \sim m_{s} / m_{b} C_{i}$, the chirality flipped operators $\mathcal{O}_{7,9,10}^{\prime}$ can only compete with $\mathcal{O}_{7,9,10}$ in models beyond the SM . The Wilson coefficients are decomposed into their SM and NP parts as $C_{i}=C_{i}^{\mathrm{SM}}+C_{i}^{\mathrm{NP}}$ and $C_{i}^{\prime}=C_{i}^{\prime \mathrm{NP}}$ for $i=7,9,10$.

## C. Transversity amplitudes at NLO

Starting from the $K^{*}$ transversity amplitudes $A_{i}\left(q^{2}\right), i=\{\perp, \|, 0, t\}$ in naive factorization (see, e.g., [9]), the NLO $\alpha_{s}$-corrections at large recoil using QCDF [12, 13] can be taken into account by the replacements [9, 10]

$$
\begin{equation*}
\left(C_{7}^{\mathrm{eff}}+C_{7}^{\prime}\right) T_{i}\left(q^{2}\right) \rightarrow \mathcal{T}_{i}^{+}, \quad\left(C_{7}^{\mathrm{eff}}-C_{7}^{\prime}\right) T_{i}\left(q^{2}\right) \rightarrow \mathcal{T}_{i}^{-}, \quad C_{9}^{\mathrm{eff}}\left(q^{2}\right) \rightarrow C_{9} \tag{C.1}
\end{equation*}
$$

where

$$
\begin{equation*}
\mathcal{T}_{1}^{ \pm}=\mathcal{T}_{\perp}^{ \pm}, \quad \mathcal{T}_{2}^{-}=\frac{2 E}{M_{B}} \mathcal{T}_{\perp}^{-}, \quad \quad \mathcal{T}_{3}^{-}=\mathcal{T}_{\perp}^{-}+\mathcal{T}_{\|}^{-} \tag{C.2}
\end{equation*}
$$

The functions $\mathcal{T}_{\perp, \|}^{-}$can be obtained from the $\mathcal{T}_{\perp, \|}$ given in 12, 13] by substituting $C_{7}^{\text {eff }}$ with $C_{7}^{\mathrm{eff}}-C_{7}^{\prime}$ whereas $\mathcal{T}_{\perp}^{+}$is obtained from $\mathcal{T}_{\perp}$ by replacing $C_{7}^{\text {eff }}$ with $C_{7}^{\mathrm{eff}}+C_{7}^{\prime}$.

In (C.1), the $T_{i}, i=1,2,3$ denote the QCD tensor form factors defined in appendix $E$. The effective electroweak Hamiltonian employed is given in appendix B. The effective coefficients $C_{7,8}^{\mathrm{eff}}$ and $C_{9}^{\mathrm{eff}}\left(q^{2}\right)$ have been introduced to absorb 1-loop matrix elements of 4-quark operators 46. Here, such contributions to $\mathcal{O}_{9}$ are contained in $\mathcal{T}_{i}^{ \pm}$together with further corrections beyond naive factorization. We take $C_{7}^{\text {eff }}$ and $C_{9,10}$ at NNLL in the SM at the scale $\mu_{b}$. In the NP scenarios discussed in this work, $C_{7}^{\prime \text { eff }}$ equals $C_{7}^{\prime}$.

In the framework of QCDF, the functions $\mathcal{T}_{\perp, \|}^{ \pm}$are known at NLO in $\alpha_{s}$ for the SM operators and the corresponding chirality flipped operators, see (B.3). The $\mathcal{T}_{\perp, \|}^{ \pm}$have the following CKM and QCD structure

$$
\begin{align*}
\mathcal{T}_{a}^{ \pm} & =\mathcal{T}_{a}^{ \pm(t)}+\hat{\lambda}_{u} \mathcal{T}_{a}^{(u)}  \tag{C.3}\\
\mathcal{T}_{a}^{ \pm(t)} & =\mathcal{T}_{a}^{ \pm(t), \mathrm{LO}}+\frac{\alpha_{s}}{4 \pi} \mathcal{T}_{a}^{ \pm(t), \mathrm{NLO}},
\end{align*} \quad \mathcal{T}_{a}^{(u)}=\mathcal{T}_{a}^{(u), \mathrm{LO}}+\frac{\alpha_{s}}{4 \pi} \mathcal{T}_{a}^{(u), \mathrm{NLO}},
$$

where $a=\perp, \|$. At LO in $\alpha_{s}$ (denoted by the superscript (0)) they read

$$
\begin{array}{ll}
\mathcal{T}_{\perp}^{ \pm(t), \mathrm{LO}}=\xi_{\perp}\left[C_{7}^{\mathrm{eff}(0)} \pm C_{7}^{\prime(0)}+\frac{q^{2}}{2 m_{b} M_{B}} Y^{(0)}\right], & \mathcal{T}_{\perp}^{(u), \mathrm{LO}}=\xi_{\perp} \frac{q^{2}}{2 m_{b} M_{B}} Y^{(u)(0)},  \tag{C.4}\\
\mathcal{T}_{\|}^{-(t), \mathrm{LO}}=-\xi_{\|}\left[C_{7}^{\mathrm{eff}(0)}-C_{7}^{\prime(0)}+\frac{M_{B}}{2 m_{b}} Y^{(0)}\right]+H S, & \mathcal{T}_{\|}^{(u), \mathrm{LO}}=-\xi_{\|} \frac{M_{B}}{2 m_{b}} Y^{(u)(0)}+H S,
\end{array}
$$

where $Y\left(q^{2}\right)$ and $Y^{(u)}\left(q^{2}\right)$ contain 1-loop contributions of four-quark operators $\sim \bar{s} b \bar{q} q$ with an imaginary part for $q^{2}>4 m_{q}^{2}$. Since the charm threshold is at the very upper end - if not outside - the $q^{2}$-region where the $1 / E$ expansion works and the lighter quarks induce either CKM suppressed or penguin contributions, the resulting strong phase is small. In (C.4), spectator effects are denoted by $H S$. At lowest order, these are in $\mathcal{T}_{\|}^{(u), \mathrm{LO}}$ and $\mathcal{T}_{\|}^{-(t), \mathrm{LO}}$. The latter is suppressed by penguin coefficients, whereas the former is non-zero only for charged $B^{ \pm} \rightarrow K^{* \pm} \bar{l} l$ decays (weak annihilation). At higher order in $\alpha_{s}$, strong phases are further generated in $\mathcal{T}_{a}^{(i), N L O}$ and from spectator interactions 12, 13], which have been included in our numerical analysis. The form factors $\xi_{\perp}$ and $\xi_{\|}$are discussed in appendix E.

The transversity amplitudes in the presence of NP Wilson coefficients within QCDF and neglecting kinematical terms ${ }^{2} M_{K^{*}}^{2} / M_{B}^{2}$ read as

$$
\begin{align*}
A_{\perp}^{L, R} & =+\sqrt{2} N M_{B}(1-\hat{s})\left\{\left[\left(C_{9}+C_{9}^{\prime}\right) \mp\left(C_{10}+C_{10}^{\prime}\right)\right] \xi_{\perp}+\frac{2 \hat{m}_{b}}{\hat{s}} \mathcal{T}_{\perp}^{+}\right\} \\
A_{\|}^{L, R} & =-\sqrt{2} N M_{B}(1-\hat{s})\left\{\left[\left(C_{9}-C_{9}^{\prime}\right) \mp\left(C_{10}-C_{10}^{\prime}\right)\right] \xi_{\perp}+\frac{2 \hat{m}_{b}}{\hat{s}} \mathcal{T}_{\perp}^{-}\right\} \\
A_{0}^{L, R} & =-\frac{N M_{B}^{2}(1-\hat{s})^{2}}{2 M_{K^{*}} \sqrt{\hat{s}}}\left\{\left[\left(C_{9}-C_{9}^{\prime}\right) \mp\left(C_{10}-C_{10}^{\prime}\right)\right] \xi_{\|}-2 \hat{m}_{b} \mathcal{T}_{\|}^{-}\right\} \\
A_{t} & =\frac{N M_{B}^{2}(1-\hat{s})^{2}}{M_{K^{*}} \sqrt{\hat{s}}}\left(C_{10}-C_{10}^{\prime}\right) \frac{\xi_{\|}}{\Delta_{\|}} \tag{C.5}
\end{align*}
$$

where

$$
\begin{equation*}
\hat{s}=\frac{q^{2}}{M_{B}^{2}}, \quad \hat{m}_{b}=\frac{m_{b}}{M_{B}}, \quad N=\left[\frac{G_{F}^{2} \alpha_{e}^{2}}{3 \cdot 2^{10} \pi^{5} M_{B}}\left|V_{t b} V_{t s}^{*}\right|^{2} \hat{s} \sqrt{\lambda} \beta_{l}\right]^{1 / 2} \tag{C.6}
\end{equation*}
$$

and $\lambda=M_{B}^{4}+M_{K^{*}}^{4}+q^{4}-2\left(M_{B}^{2} M_{K^{*}}^{2}+M_{B}^{2} q^{2}+M_{K^{*}}^{2} q^{2}\right)$. Note that $A_{t}$ contributes only for $m_{l} \neq 0$ and contains $\Delta_{\|}$, see 12, which represents form factor symmetry breaking QCD corrections. Note that helicity conservation dictates $A_{\perp}^{L, R}=-A_{\|}^{L, R}$ for $C_{i}^{\prime}=0$ up to $1 / E$ corrections 47].

The dilepton spectrum defined in (2.7) can be written in terms of the transversity amplitudes (C.5) as

$$
\begin{equation*}
\frac{d \Gamma}{d q^{2}}=\left|A_{\perp}^{L}\right|^{2}+\left|A_{\|}^{L}\right|^{2}+\left|A_{0}^{L}\right|^{2}+(L \rightarrow R)+\mathcal{O}\left(m_{l}^{2} / q^{2}\right) \tag{C.7}
\end{equation*}
$$

[^1]up to contributions suppressed by the lepton mass. The dependence on the form factors stems from $\xi_{\perp}$ for $A_{\perp}^{L, R}, A_{\|}^{L, R}$ and $\xi_{\|}$for $A_{0}^{L, R}$. Since the longitudinal amplitudes $A_{0}^{L, R}$ are enhanced by $M_{B} / M_{K^{*}}$, see (C.5), they imply a stronger dependence of $d \Gamma / d q^{2}$ on $\xi_{\|}$than on $\xi_{\perp}$. Quantitatively, we find in the SM that for the cuts $\left(q_{\min }^{2}, q_{\max }^{2}\right)=(1,6) \mathrm{GeV}^{2}$ and $(1,7) \mathrm{GeV}^{2}$ the contribution from the longitudinal $K^{*}$ to the total decay rate,
\[

$$
\begin{equation*}
\mathcal{F}_{L}=\frac{\left.\left.\langle | A_{0}^{L}\right|^{2}+\left|A_{0}^{R}\right|^{2}\right\rangle}{\left\langle d \Gamma / d q^{2}\right\rangle}, \tag{C.8}
\end{equation*}
$$

\]

is $0.73_{-0.10}^{+0.08}$ and $0.72_{-0.11}^{+0.08}$, respectively.

## D. CP asymmetries and $A_{\text {FB }}$ beyond the SM

Here we give analytical expressions of the CP asymmetries defined in (3.2) and (3.3) including contributions from NP operators (B.3). The asymmetries have been obtained from the transversity amplitudes in QCDF, see (C.5), valid in the low dilepton mass region. The coefficients $C_{7}^{\mathrm{eff}}=C_{7}^{\mathrm{eff}, \mathrm{SM}}+C_{7}^{\mathrm{NP}}$ and $C_{7}^{\prime}$ are taken into account by $\mathcal{T}_{\perp, \|}^{ \pm}$. Except for $A_{\mathrm{CP}}$, the CP asymmetries are given with their full lepton mass dependence which is confined to powers of $\beta_{l}$. Neglecting kinematical factors $M_{K^{*}}^{2} / M_{B}^{2}$, the CP asymmetries as a function of the dilepton mass can be written as

$$
\begin{align*}
& A_{\mathrm{CP}}=\mathcal{A} \frac{8 \hat{m}_{b}}{3 \hat{s}} \operatorname{Re}\left\{\frac{\xi_{\|}^{2}}{\xi_{\perp}^{2}} \frac{M_{B}^{2}}{M_{K^{*}}^{2}} \frac{(1-\hat{s})^{2}}{8}\left[\hat{m}_{b} \frac{\left|\mathcal{T}_{\|}^{-}\right|^{2}}{\xi_{\|}^{2}}-\frac{\mathcal{T}_{\|}^{-}}{\xi_{\|}}\left(C_{9}-C_{9}^{\prime}\right)^{*}\right]+\frac{\hat{m}_{b}\left|\mathcal{T}_{\perp}^{+}\right|^{2}+\left|\mathcal{T}_{\perp}^{-}\right|^{2}}{\xi_{\perp}^{2}} \frac{\mathcal{T}_{\perp}}{\mathcal{T}^{2}}\right. \\
& \left.+\frac{\mathcal{T}_{\perp}^{+}-\mathcal{T}_{\perp}^{-}}{\xi_{\perp}} C_{9}^{*}+\frac{\mathcal{T}_{\perp}^{+}+\mathcal{T}_{\perp}^{-}}{\xi_{\perp}} C_{9}^{\prime *}-\left(\delta_{W} \rightarrow-\delta_{W}\right)\right\}+\mathcal{O}\left(m_{l}^{2} / q^{2}\right),  \tag{D.1}\\
& A_{3}=\mathcal{A} \frac{2 \hat{m}_{b} \beta_{l}}{\hat{s}} \operatorname{Re}\left\{\frac{\hat{m}_{b}}{\hat{s}} \frac{\left|\mathcal{T}_{\perp}^{+}\right|^{2}-\left|\mathcal{T}_{\perp}^{-}\right|^{2}}{\xi_{\perp}^{2}}+\frac{\mathcal{T}_{\perp}^{+}-\mathcal{T}_{\perp}^{-}}{\xi_{\perp}} C_{9}^{*}+\frac{\mathcal{T}_{\perp}^{+}+\mathcal{T}_{\perp}^{-}}{\xi_{\perp}} C_{9}^{\prime *}-\left(\delta_{W} \rightarrow-\delta_{W}\right)\right\},  \tag{D.2}\\
& A_{4}^{D}=-\mathcal{A}^{D} \frac{\hat{m}_{b} \beta_{l}}{2 \hat{s}} \operatorname{Re}\left\{\left(\frac{\mathcal{T}_{\perp}^{-}}{\xi_{\perp}}-\hat{s} \frac{\mathcal{T}_{\|}^{-}}{\xi_{\|}}\right)\left(C_{9}-C_{9}^{\prime}\right)^{*}-2 \hat{m}_{b} \frac{\mathcal{T}_{\perp}^{-}\left(\mathcal{T}_{\|}^{-}\right)^{*}}{\xi_{\perp} \xi_{\|}}-\left(\delta_{W} \rightarrow-\delta_{W}\right)\right\},  \tag{D.3}\\
& A_{5}^{D}=-\mathcal{A}^{D} \frac{\hat{m}_{b}}{\hat{s}} \operatorname{Re}\left\{\left(\frac{\mathcal{T}_{\perp}^{-}}{\xi_{\perp}}-\hat{s} \frac{\mathcal{T}_{\|}^{-}}{\xi_{\|}}\right) C_{10}-\left(\frac{\mathcal{T}_{\perp}^{-}}{\xi_{\perp}}+\hat{s} \frac{\mathcal{T}_{\|}^{-}}{\xi_{\|}}\right) C_{10}^{\prime *}-\left(\delta_{W} \rightarrow-\delta_{W}\right)\right\},  \tag{D.4}\\
& A_{6}=\mathcal{A} \frac{4 \hat{m}_{b}}{\hat{s}} \operatorname{Re}\left\{\frac{\mathcal{T}_{\perp}^{+}+\mathcal{T}_{\perp}^{-}}{\xi_{\perp}} C_{10}^{*}-\frac{\mathcal{T}_{\perp}^{+}-\mathcal{T}_{\perp}^{-}}{\xi_{\perp}} C_{10}^{* *}-\left(\delta_{W} \rightarrow-\delta_{W}\right)\right\},  \tag{D.5}\\
& A_{7}^{D}=\mathcal{A}^{D} \frac{\hat{m}_{b}}{\hat{s}} \operatorname{Im}\left\{\left(C_{10}-C_{10}^{\prime}\right)\left(\frac{\mathcal{T}_{\perp}^{-}}{\xi_{\perp}}+\hat{s} \frac{\mathcal{T}_{\|}^{-}}{\xi_{\|}}\right)^{*}-\left(\delta_{W} \rightarrow-\delta_{W}\right)\right\},  \tag{D.6}\\
& A_{8}^{D}=\mathcal{A}^{D} \frac{\beta_{l}}{2} \operatorname{Im}\left\{\frac{2 \hat{m}_{b}^{2}}{\hat{s}} \frac{\mathcal{T}_{\perp}^{+}\left(\mathcal{T}_{\|}^{-}\right)^{*}}{\xi_{\perp} \xi_{\|}}-\frac{\hat{m}_{b}}{\hat{s}}\left[\left(\frac{\mathcal{T}_{\perp}^{+}}{\xi_{\perp}}+\hat{s} \frac{\mathcal{T}_{\|}^{-}}{\xi_{\|}}\right) C_{9}^{*}-\left(\frac{\mathcal{T}_{\perp}^{+}}{\xi_{\perp}}-\hat{s} \frac{\mathcal{T}_{\|}^{-}}{\xi_{\|}}\right) C_{9}^{\prime *}\right]\right. \\
& \left.+C_{9} C_{9}^{\prime *}+C_{10} C_{10}^{\prime *}-\left(\delta_{W} \rightarrow-\delta_{W}\right)\right\}, \tag{D.7}
\end{align*}
$$

$$
\begin{align*}
A_{9}=-\mathcal{A} 2 \beta_{l} \operatorname{Im}\{ & \frac{2 \hat{m}_{b}^{2}}{\hat{s}^{2}} \frac{\mathcal{T}_{\perp}^{+}\left(\mathcal{T}_{\perp}^{-}\right)^{*}}{\xi_{\perp}^{2}}+\frac{\hat{m}_{b}}{\hat{s}}\left[\frac{\mathcal{T}_{\perp}^{+}-\mathcal{T}_{\perp}^{-}}{\xi_{\perp}} C_{9}^{*}-\frac{\mathcal{T}_{\perp}^{+}+\mathcal{T}_{\perp}^{-}}{\xi_{\perp}} C_{9}^{\prime *}\right] \\
& \left.-C_{9} C_{9}^{* *}-C_{10} C_{10}^{* *}-\left(\delta_{W} \rightarrow-\delta_{W}\right)\right\}, \tag{D.8}
\end{align*}
$$

where $\left(\delta_{W} \rightarrow-\delta_{W}\right)$ is short hand notation for conjugating all weak phases. Furthermore,

$$
\begin{align*}
\mathcal{A} & =\frac{G_{F}^{2} \alpha_{e}^{2}}{3^{2} \cdot 2^{6} \pi^{5}}\left|V_{t b} V_{t s}^{*}\right|^{2} \frac{M_{B}^{3} \beta_{l}^{2} \hat{s}(1-\hat{s})^{3} \xi_{\perp}^{2}}{N_{\Gamma}}, \\
\mathcal{A}^{D} & =\frac{G_{F}^{2} \alpha_{e}^{2}}{3^{2} \cdot 2^{6} \pi^{5}}\left|V_{t b} V_{t s}^{*}\right|^{2} \frac{M_{B}^{4} \beta_{l}^{2} \sqrt{\hat{s}}(1-\hat{s})^{4} \xi_{\|} \xi_{\perp}}{M_{K^{*}} N_{\Gamma}}, \tag{D.9}
\end{align*}
$$

where $N_{\Gamma}$ is defined in (3.2).
At lowest order in $\alpha_{s}$, the expressions for the above CP asymmetries simplify by

$$
\begin{align*}
& \frac{\mathcal{T}_{\perp}^{+, \mathrm{LO}}-\mathcal{T}_{\perp}^{-, \mathrm{LO}}}{\xi_{\perp}}=2 C_{7}^{\prime(0)},  \tag{D.10}\\
& \frac{\mathcal{T}_{\perp}^{+, \mathrm{LO}}+\mathcal{T}_{\perp}^{-, \mathrm{LO}}}{\xi_{\perp}}=2 C_{7}^{\mathrm{eff}(0)}+\frac{\hat{s}}{\hat{m}_{b}}\left(Y^{(0)}+\hat{\lambda}_{u} Y^{(u)(0)}\right),  \tag{D.11}\\
& \frac{\mathcal{T}_{\perp}^{ \pm, \mathrm{LO}}}{\xi_{\perp}}+\hat{s} \frac{\mathcal{T}_{\|}^{-, \mathrm{LO}}}{\xi_{\|}}=\left\{\begin{array}{c}
(1-\hat{s}) C_{7}^{\mathrm{eff}(0)}+(1+\hat{s}) C_{7}^{\prime(0)} \\
(1-\hat{s})\left(C_{7}^{\mathrm{eff}(0)}-C_{7}^{\prime(0)}\right)
\end{array}\right. \tag{D.12}
\end{align*}
$$

Note that in the SM, or more general, in any model without right-handed contributions to the electromagnetic dipole operator, $\mathcal{T}_{\perp}^{+}=\mathcal{T}_{\perp}^{-}$, see appendix $\mathbb{G}$.

The lepton forward-backward asymmetry in QCDF is written as

$$
\begin{align*}
& A_{\mathrm{FB}}= \frac{12 \beta_{l} N^{2} M_{B}^{2}(1-\hat{s})^{2} \xi_{\perp}^{2}}{d \Gamma / d q^{2}}  \tag{D.13}\\
& \quad \times \operatorname{Re}\left\{\left[C_{9}+\frac{\hat{m}_{b}}{\hat{s}} \frac{\left(\mathcal{T}_{\perp}^{+}+\mathcal{T}_{\perp}^{-}\right)}{\xi_{\perp}}\right] C_{10}^{*}-\left[C_{9}^{\prime}+\frac{\hat{m}_{b}}{\hat{s}} \frac{\left(\mathcal{T}_{\perp}^{+}-\mathcal{T}_{\perp}^{-}\right)}{\xi_{\perp}}\right] C_{10}^{\prime *}\right\} .
\end{align*}
$$

## E. $B \rightarrow K^{*}$ form factors at large recoil

The $B \rightarrow K^{*}$ matrix element can be parametrized in terms of seven $q^{2}$-dependent QCD form factors $V, A_{0,1,2}$ and $T_{1,2,3}$ as

$$
\begin{align*}
& \left\langle K^{*}\left(p_{B}-q\right)\right| \bar{s} \gamma_{\mu}\left(1-\gamma_{5}\right) b\left|B\left(p_{B}\right)\right\rangle=-2 \epsilon_{\mu \nu \alpha \beta} \varepsilon^{* \nu} p_{B}^{\alpha} q^{\beta} \frac{V}{M_{B}+M_{K^{*}}}  \tag{E.1}\\
& \quad-i \varepsilon_{\mu}^{*}\left(M_{B}+M_{K^{*}}\right) A_{1}+i\left(2 p_{B}-q\right)_{\mu}\left(\varepsilon^{*} \cdot q\right) \frac{A_{2}}{M_{B}+M_{K^{*}}}+i q_{\mu}\left(\varepsilon^{*} \cdot q\right) \frac{2 M_{K^{*}}}{q^{2}}\left[A_{3}-A_{0}\right], \\
& \left\langle K^{*}\left(p_{B}-q\right)\right| \bar{s} \sigma_{\mu \nu} q^{\nu}\left(1+\gamma_{5}\right) b\left|B\left(p_{B}\right)\right\rangle=-2 i \epsilon_{\mu \nu \alpha \beta} \varepsilon^{* \nu} p_{B}^{\alpha} q^{\beta} T_{1}  \tag{E.2}\\
& \quad+\left[\varepsilon_{\mu}^{*}\left(M_{B}^{2}-M_{K^{*}}^{2}\right)-\left(\varepsilon^{*} \cdot q\right)\left(2 p_{B}-q\right)_{\mu}\right] T_{2}+\left(\varepsilon^{*} \cdot q\right)\left[q_{\mu}-\frac{q^{2}}{M_{B}^{2}-M_{K^{*}}^{2}}\left(2 p_{B}-q\right)_{\mu}\right] T_{3}
\end{align*}
$$

|  | $r_{1}$ | $r_{2}$ | $m_{R}^{2}\left[\mathrm{GeV}^{2}\right]$ | $m_{\text {fit }}^{2}\left[\mathrm{GeV}^{2}\right]$ | $F(0)$ | $\Delta_{0} F(0)$ | $\Delta_{a_{1}} F(0)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $V$ | 0.923 | -0.511 | $5.32^{2}$ | 49.40 | 0.411 | 0.033 | $0.44 \delta_{a_{1}}$ |
| $A_{1}$ |  | 0.290 |  | 40.38 | 0.292 | 0.028 | $0.33 \delta_{a_{1}}$ |
| $A_{2}$ | -0.084 | 0.342 |  | 52.00 | 0.259 | 0.027 | $0.31 \delta_{a_{1}}$ |

Table 5: The parameters $r_{1,2}, m_{R}^{2}$ and $m_{\text {fit }}^{2}$ describing the $q^{2}$ dependence of the form factors $V$ and $A_{1,2}$ in the LCSR approach [25]. Also shown are the corresponding values of the form factors at $q^{2}=0, F(0)$, their uncertainties independent of the Gegenbauer moment $a_{1, K^{*}}, \Delta_{0} F(0)$ and the uncertainties induced by $a_{1, K^{*}}$ in terms of $\delta_{a_{1}}=\left(a_{1, K^{*}}(1 \mathrm{GeV})-0.1\right), \Delta_{a_{1}} F(0)$.
and

$$
\begin{equation*}
A_{3}=\frac{M_{B}+M_{K^{*}}}{2 M_{K^{*}}} A_{1}-\frac{M_{B}-M_{K^{*}}}{2 M_{K^{*}}} A_{2}, \tag{E.3}
\end{equation*}
$$

where $\epsilon^{* \mu}$ denotes the polarization vector of the $K^{*}$ and $p_{B}^{\mu}$ the four momentum of the $B$ meson. The QCD form factors obey symmetry relations in the large recoil limit and can be expressed at leading order in the $1 / E$ expansion in terms of two universal form factors $\xi_{\perp}$ and $\xi_{\|}$11]. Symmetry breaking corrections at order $\alpha_{s}$ have been calculated using QCDF in ref. 48]. We employ a factorization scheme within QCDF where the $\xi_{\perp, \|}$ are related to the $V, A_{1,2}$ as (13]

$$
\begin{equation*}
\xi_{\perp}=\frac{M_{B}}{M_{B}+M_{K^{*}}} V, \quad \xi_{\|}=\frac{M_{B}+M_{K^{*}}}{2 E} A_{1}-\frac{M_{B}-M_{K^{*}}}{M_{B}} A_{2} \tag{E.4}
\end{equation*}
$$

For the $q^{2}$ dependence of the universal form factors we adopt the findings from light cone sum rule (LCSR) calculations (25). Here the $q^{2}$ dependence is parametrized as

$$
\begin{align*}
V\left(q^{2}\right) & =\frac{r_{1}}{1-q^{2} / m_{R}^{2}}+\frac{r_{2}}{1-q^{2} / m_{\mathrm{fit}}^{2}},  \tag{E.5}\\
A_{1}\left(q^{2}\right) & =\frac{r_{2}}{1-q^{2} / m_{\mathrm{fit}}^{2}},  \tag{E.6}\\
A_{2}\left(q^{2}\right) & =\frac{r_{1}}{1-q^{2} / m_{\mathrm{fit}}^{2}}+\frac{r_{2}}{\left(1-q^{2} / m_{\mathrm{fit}}^{2}\right)^{2}}, \tag{E.7}
\end{align*}
$$

where the fit parameters $r_{1,2}, m_{R}^{2}$ and $m_{\text {fit }}^{2}$ are shown in table 0 . Also given in this table are the values of the form factors at $q^{2}=0$ and the corresponding parametric uncertainties within the LCSR approach. We give the uncertainties independent of the Gegenbauer moments $a_{1, K^{*}}^{\perp, \|}$ and the ones due to $a_{1, K^{*}}^{\perp, \|}$ separately. The relative uncertainty of the form factors $V(0), A_{1}(0)$ and $A_{2}(0)$ amounts to $8 \%, 10 \%$ and $10 \%$ without, and $11 \%, 12 \%$ and $14 \%$ after adding the $a_{1, K^{*}}$ - see table 1 for the numerical value - induced uncertainty in quadrature, respectively. We use the total relative uncertainty from maximal recoil as an estimate for the form factor uncertainties for $q^{2}>0$. The form factors $\xi_{\perp, \|}$ defined via (E.4) are shown as a function of $q^{2}$ in figure 5 . Here the bands indicate the uncertainty in $\xi_{\perp}$ and $\xi_{\|}$of $11 \%$ and $14 \%$, respectively.

## F. Model-independent CP asymmetries beyond the SM

We give numerical formulae for the $q^{2}$-integrated quantities $\mathcal{B}=\tau_{B^{0}}\left\langle d \Gamma / d q^{2}\right\rangle, \overline{\mathcal{B}}=$ $\tau_{B^{0}}\left\langle d \bar{\Gamma} / d q^{2}\right\rangle$ and Num $\left\langle A_{i}^{(D)}\right\rangle$ for $q^{2} \in[1,6] \mathrm{GeV}^{2}$ in terms of the NP Wilson coefficients


Figure 5: The universal form factors $\xi_{\perp}$ and $\xi_{\|}$in the low- $q^{2}$ region and their uncertainty indicated by the bands.
$C_{i}^{\text {NP }}$. Here, $\operatorname{Num}\left\langle A_{i}^{(D)}\right\rangle$ denotes the numerators of the CP asymmetries multiplied by the $B$-meson lifetime such that the normalized CP asymmetries (see (3.5)) are obtained from

$$
\begin{equation*}
\left\langle A_{i}^{(D)}\right\rangle=\frac{\operatorname{Num}\left\langle A_{i}^{(D)}\right\rangle}{\mathcal{B}+\overline{\mathcal{B}}} \tag{F.1}
\end{equation*}
$$

The dependence of the branching ratios on the NP Wilson coefficients can be written as

$$
X=X_{\mathrm{SM}}\left[1+\sum_{i}\left(a_{i}\left|C_{i}^{\mathrm{NP}}\right|^{2}+b_{i} \operatorname{Re} C_{i}^{\mathrm{NP}}+c_{i} \operatorname{Im} C_{i}^{\mathrm{NP}}\right)+\sum_{j>i} d_{i j} \operatorname{Re}\left(C_{i}^{\mathrm{NP}} C_{j}^{\mathrm{NP} *}\right)\right] \quad \text { for } \mathcal{B}, \overline{\mathcal{B}}, \quad(\mathrm{F} .2)
$$

whereas the numerators of the T-odd CP asymmetries are parametrized as

$$
\begin{equation*}
X=X_{\mathrm{SM}}\left[1+\sum_{i}\left(b_{i} \operatorname{Re} C_{i}^{\mathrm{NP}}+c_{i} \operatorname{Im} C_{i}^{\mathrm{NP}}\right)+\sum_{j>i} e_{i j} \operatorname{Im}\left(C_{i}^{\mathrm{NP}} C_{j}^{\mathrm{NP} *}\right)\right] \text { for } \operatorname{Num}\left\langle A_{7,8}^{D}\right\rangle \tag{F.3}
\end{equation*}
$$

The numerators of the T-even CP asymmetries read as

$$
\begin{equation*}
X=X_{\mathrm{SM}}\left[1+\sum_{i}\left(b_{i} \operatorname{Re} C_{i}^{\mathrm{NP}}+c_{i} \operatorname{Im} C_{i}^{\mathrm{NP}}\right)\right] \quad \text { for } \operatorname{Num}\left\langle A_{\mathrm{CP}, 6}\right\rangle, \operatorname{Num}\left\langle A_{4,5}^{D}\right\rangle . \tag{F.4}
\end{equation*}
$$

Here, the summations are over $i, j=7,7^{\prime}, 9,9^{\prime}, 10,10^{\prime}$ and $X_{\text {SM }}$ denotes the SM prediction of the corresponding quantity. Note that for $\operatorname{Num}\left\langle A_{3,9}\right\rangle$ we have set $X_{\text {SM }}$ to zero, see section 國, and, hence, the corresponding formulae read as

$$
\begin{align*}
& X=\sum_{i}\left(b_{i} \operatorname{Re} C_{i}^{\mathrm{NP}}+c_{i} \operatorname{Im} C_{i}^{\mathrm{NP}}\right) \quad \text { for } \operatorname{Num}\left\langle A_{3}\right\rangle,  \tag{F.5}\\
& X=\sum_{i}\left(b_{i} \operatorname{Re} C_{i}^{\mathrm{NP}}+c_{i} \operatorname{Im} C_{i}^{\mathrm{NP}}\right)+\sum_{j>i} e_{i j} \operatorname{Im}\left(C_{i}^{\mathrm{NP}} C_{j}^{\mathrm{NP} *}\right) \quad \text { for } \operatorname{Num}\left\langle A_{9}\right\rangle . \tag{F.6}
\end{align*}
$$

| $X_{\mathrm{SM}}$ |  | $i=7$ | $i=7^{\prime}$ | $i=9$ | $i=9^{\prime}$ | $i=10$ | $i=10^{\prime}$ |
| :---: | :---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $\mathcal{B}_{\mathrm{SM}}$ | $a_{i}$ | 2.634 | 2.634 | 0.035 | 0.035 | 0.035 | 0.035 |
| $=2.444 \cdot 10^{-7}$ | $b_{i}$ | -0.271 | -0.373 | 0.162 | -0.179 | -0.288 | 0.205 |
| $c_{i}$ | -0.156 | 0.001 | -0.009 | -0.0002 | 0 | 0 |  |
| $\overline{\mathcal{B}}_{\text {SM }}$ | $a_{i}$ | 2.656 | 2.656 | 0.036 | 0.036 | 0.035 | 0.035 |
| $=2.423 \cdot 10^{-7}$ | $b_{i}$ | $c_{i}$ | -0.312 | -0.370 | 0.158 | -0.178 | -0.290 |
| Num $\left\langle A_{\mathrm{CP}}\right\rangle_{\mathrm{SM}}$ | $b_{i}$ | 4.469 | -0.003 | 0.004 | 0.002 | 0 | 0 |
| $=2.068 \cdot 10^{-9}$ | $c_{i}$ | -30.770 | -0.275 | -1.587 | -0.345 | 0 | 0 |
| Num $\left\langle A_{3}\right\rangle_{\mathrm{SM}}$ | $b_{i}$ | -0.077 | 5.720 | -0.012 | -0.259 | 0 | 0 |
| $=0^{\dagger}$ | $c_{i}$ | 0.542 | -47.174 | 0.081 | -2.748 | 0 | 0 |
| Num $\left\langle A_{4}^{D}\right\rangle_{\mathrm{SM}}$ | $b_{i}$ | 3.604 | -3.604 | 0.536 | -0.536 | 0 | 0 |
| $=-8.642 \cdot 10^{-10}$ | $c_{i}$ | -1.435 | 1.435 | -2.487 | 2.487 | 0 | 0 |
| Num $\left\langle A_{5}^{D}\right\rangle_{\mathrm{SM}}$ | $b_{i}$ | 0 | 0 | 0 | 0 | -0.244 | 0.068 |
| $=3.718 \cdot 10^{-9}$ | $c_{i}$ | 0 | 0 | 0 | 0 | 1.152 | -1.258 |
| Num $\left\langle A_{6}\right\rangle_{\mathrm{SM}}$ | $b_{i}$ | 0 | 0 | 0 | 0 | -0.244 | 0.004 |
| $=-3.117 \cdot 10^{-9}$ | $c_{i}$ | 0 | 0 | 0 | 0 | 1.774 | -0.026 |
| Num $\left\langle A_{7}^{D}\right\rangle_{\mathrm{SM}}$ | $b_{i}$ | 0 | 0 | 0 | 0 | -0.244 | 0.244 |
| $=-2.496 \cdot 10^{-9}$ | $c_{i}$ | -247.248 | 247.248 | 0 | 0 | 23.019 | -23.019 |
| Num $\left\langle A_{8}^{D}\right\rangle_{\mathrm{SM}}$ | $b_{i}$ | -0.491 | -1.423 | 0.176 | -0.288 | 0 | 0 |
| $=1.706 \cdot 10^{-9}$ | $c_{i}$ | -189.333 | -170.364 | -16.524 | -7.160 | 0 | 26.834 |
| Num $\left\langle A_{9}\right\rangle_{\mathrm{SM}}$ | $b_{i}$ | 0 | -8.390 | 0.007 | -0.491 | 0 | 0 |
| $=0^{\dagger}$ | $c_{i}$ | -6.514 | 225.487 | -0.568 | 6.064 | 0 | 31.913 |

Table 6: The SM predictions $X_{\mathrm{SM}}$ and the corresponding coefficients $a_{i}, b_{i}$ and $c_{i}$ for $i=$ $7,7^{\prime}, 9,9^{\prime}, 10,10^{\prime}$. ${ }^{\dagger}$ For Num $\left\langle A_{3,9}\right\rangle X_{\mathrm{SM}}$ has been set to zero and the corresponding coefficients are given in units of $10^{-9}$.

| $d_{i j}$ | $\mathcal{B}$ | $\overline{\mathcal{B}}$ | $e_{i j}$ | $\operatorname{Num}\left\langle A_{7}^{D}\right\rangle$ | $\operatorname{Num}\left\langle A_{8}^{D}\right\rangle$ | $\operatorname{Num}\left\langle A_{9}\right\rangle^{\dagger}$ |
| :---: | ---: | ---: | :---: | ---: | ---: | ---: |
| $7,7^{\prime}$ | -0.255 | -0.257 | $7,7^{\prime}$ | 0 | 200.542 | 1801.269 |
| 7,9 | 0.394 | 0.397 | 7,9 | 0 | -43.413 | -1.547 |
| $7,9^{\prime}$ | -0.107 | -0.108 | $7,9^{\prime}$ | 0 | 56.532 | 105.869 |
| 7,10 | 0 | 0 | 7,10 | 60.420 | 0 | 0 |
| $7,10^{\prime}$ | 0 | 0 | $7,10^{\prime}$ | -60.420 | 0 | 0 |
| $7^{\prime}, 9$ | -0.107 | -0.108 | $7^{\prime}, 9$ | 0 | -56.532 | -105.869 |
| $7^{\prime}, 9^{\prime}$ | 0.394 | 0.397 | $7^{\prime}, 9^{\prime}$ | 0 | 43.413 | 1.547 |
| $7^{\prime}, 10$ | 0 | 0 | $7^{\prime}, 10$ | -60.420 | 0 | 0 |
| $7^{\prime}, 10^{\prime}$ | 0 | 0 | $7^{\prime}, 10^{\prime}$ | 60.420 | 0 | 0 |
| $9,9^{\prime}$ | -0.050 | -0.050 | $9,9^{\prime}$ | 0 | 6.558 | 7.799 |
| $10,10^{\prime}$ | -0.050 | -0.050 | $10,10^{\prime}$ | 0 | 6.558 | 7.799 |

Table 7: The coefficients $d_{i j}$ and $e_{i j}$ for $i, j=7,7^{\prime}, 9,9^{\prime}, 10,10^{\prime}$ and $j>i$. $^{\dagger}$ For Num $\left\langle A_{9}\right\rangle X_{\mathrm{SM}}$ has been set to zero and the corresponding coefficients are given in units of $10^{-9}$.

The SM predictions $X_{\text {SM }}$ and the coefficients $a_{i}, b_{i}, c_{i}$ and $d_{i j}, e_{i j}$ are given in table 6 and table 7 , respectively. We assumed central values for all parameters.

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[^0]:    ${ }^{1}$ Since we define the lepton angle $\theta_{l}$ with respect to the $l^{-}$, our definiton of the forward-backward asymmetry (2.8) differs from the one in other works using the $l^{+}$, e.g., 用, 道, 12, 13, 21, by a global sign.

[^1]:    ${ }^{2}$ These formally subleading terms in the $1 / E$ expansion are included in the numerical evaluation.

